

# Extended thermodynamics of heat transport and energy equilibration in radiative systems

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**Abstract.** The assumptions of a simple thermodynamical theory of radiative transfer are modified, and consistency with some recent statistical-mechanical results is achieved. The model presented provides a thermodynamical framework for the description not only of radiative heat transport but also of energy exchange between matter and radiation.

## 1. Introduction

Some time ago, we proposed a phenomenological approach to nonequilibrium thermodynamics of radiative transfer [1]. By radiative transfer we mean situations such that heat is transported by radiation (photons), whereas conduction and convection are negligible: the conductive heat flux is assumed to be negligible, and the matter is assumed to be at rest in order to avoid convective effects. That model was based on the simplest assumptions possible, which we now summarize:

(i) In near-equilibrium states, the specific entropy depends only on the specific energy. By analogy to local-equilibrium thermodynamics of purely matter systems [2], we denoted such states as local-equilibrium states. In this case,  $s_{le} = s_{le}(u)$ , where  $s_{le}$  is the local-equilibrium entropy per unit mass and  $u$  is the energy per unit mass of the system composed by matter and radiation, i.e.

$$u = u_m + u_r \quad (1)$$

where  $u_m$  is the internal energy of matter per unit mass, and  $u_r$  the radiation energy, also per unit mass of matter. In local equilibrium, the temperature  $T$  of the system was defined thermodynamically as (see equation (11) in [1])

$$\frac{1}{T} \equiv \frac{ds_{le}(u)}{du}. \quad (2)$$

(ii) From the following evolution equation, which was derived on phenomenological grounds [3, 1],

$$\tau_r \frac{d\vec{F}}{dt} + \vec{F} = -\lambda_r \vec{\nabla} T \quad (3)$$

where  $\tau_r$  is a relaxation time,  $\vec{F}$  is the radiative energy flux and  $\lambda_r$  may be called the radiative conductivity, it was shown [1] that assumption (i) is not compatible with the second law of thermodynamics. Thus we also assumed (in analogy to extended thermodynamics of purely matter systems [4]) that, in general, the specific entropy may depend on the radiative flux  $\vec{F}$  in addition to the specific energy, i.e.  $s = s(u, \vec{F})$ . A generalized temperature  $\theta$  was defined through

$$\frac{1}{\theta} \equiv \frac{\partial s(u, \vec{F})}{\partial u} \quad (4)$$

where  $u$  is given by equation (1). Under those assumptions, it was shown in [1] that  $\theta$  is related to the local-equilibrium temperature  $T$  of equation (2) as follows:

$$\frac{1}{\theta} = \frac{1}{T} - \frac{\vec{F} \cdot \vec{F}}{2} \frac{\partial}{\partial u} \left( \frac{\tau_r}{\rho \lambda_r T^2} \right) \quad (5)$$

where  $\rho$  is the matter density.

In section 2 we show that the model presented in [1], and summarized above, is not sufficiently general and does not yield rigorous consistency with microscopic results. In section 3, we develop the basis of a more general model by making use of an approach inspired by the methods of classical irreversible thermodynamics [2]. In section 4, we extend those results by means of an analysis along the lines of extended irreversible thermodynamics [4]. The approach presented also takes proper account of the process of energy equilibration between radiation and matter. Section 5 is devoted to the proof of consistency with the less general, though more rigorous, microscopic results developed so far. Finally, in section 6 we include some concluding remarks.

## 2. Critical remarks on previous work

The model based on assumptions (i) and (ii) recalled in the introduction provided a useful first step and yielded reasonable results, but we think that it should now be reconsidered: here we will argue that assumptions (i) and (ii) (which were the starting point in [1]) cannot yield a reasonably general and fully consistent model, because of the following points.

(a) In general, there is no reason to assume that the system may be assigned a single temperature field, as done in equations (2) and (4). For example, if the radiation incident on the system is increased, the absorption of radiation by matter will also increase. This will cause a higher matter temperature, which in turn enhances the emission of radiation by matter. Both process (emission and absorption) will eventually balance if the radiative field incident on the system is constant. Only in this case will matter and radiation become thermalized. However, even in such a case, until the final steady state is attained there is no reason to assign a single temperature to matter and radiation. It is also clear that if the timescale of the variations of the incident radiation is faster than that of the thermalization process, then a sensible thermodynamical description should not be based on the simple assumption of a common temperature.

(b) Equation (5) was derived phenomenologically in [1]. Because the results described above were based on phenomenological assumptions, we have recently presented a microscopic, more rigorous analysis. In the original work<sup>†</sup>, this analysis was applied to

<sup>†</sup> In [5], the symbol  $s$  was used to denote entropy densities. In contrast, in this paper we follow the standard thermodynamical notation in which  $s$  refers to specific entropies (i.e., entropies per unit mass).

steady states. The application to non-steady states has been presented in [6]. The starting point of the formalism is the microscopic expression for the entropy density, namely [7]

$$\rho s = \rho s_m + \rho s_r = -k \int \frac{d^3 p_m}{h^3} f_m \ln f_m + 2k \int \frac{d^3 p_r}{h^3} [(1 + f_r) \ln(1 + f_r) - f_r \ln f_r] \quad (6)$$

where the subscripts  $m$  and  $r$  refer to matter and radiation;  $f$  and  $\vec{p}$  are the corresponding momentum distribution functions and momenta, respectively;  $k$  is the Boltzmann constant and  $h$  is the Planck constant. The expression for  $\rho s_m$  in equation (6) holds for a classical ideal gas, though the results of the theory in [5, 6] have been recently shown to also hold for other kinds of matter systems [8]. Here it is sufficient to say that, within this procedure, one makes use of information statistical theory in order to determine  $f_r$  and  $f_m$  and, after insertion of  $f_r$  and  $f_m$  into equation (6), one derives expressions for  $s_m$  and  $s_r$ , which are of the form

$$s = s_m(u_m) + s_r(u_r, \vec{F}) \quad (7)$$

from which one may evaluate the entropy differential  $ds$  in terms of macroscopic variables; use of the definition (4) then yields an expression for  $\theta$ , namely†

$$\frac{1}{\theta} = \frac{1}{T_r} - \frac{\vec{F} \cdot \vec{F}}{2} \frac{\partial}{\partial u_r} \left( \frac{\tau_r}{\rho \lambda_r T_r^2} \right) \quad (8)$$

where  $T_r$  can be written as [9]

$$\frac{1}{T_r} \equiv \frac{\partial s_{le}(u_m, u_r, \vec{F})}{\partial u_r} = \frac{\partial s_r^{le}(u_r, \vec{F})}{\partial u_r} \quad (9)$$

and the subscript and superscript  $le$  refer to states such that  $s_r$  does not depend on  $\vec{F}$  but only on  $u_r$  [9] (see equation (7)). The last equality in equation (9) follows from equation (7).

A problem arises when one notes that the microscopically-derived equation (8) is different from the phenomenological result (5). But equation (5) is based, as summarized above, on the assumption that matter and radiation share a single temperature. This is precisely the assumption we have criticized in point (a). Thus it seems sensible to try to solve both problems (a) and (b) by considering that in general, matter and radiation do not share a simple temperature. This is the approach followed in this paper.

### 3. Local equilibrium

From the considerations in the introduction it is clear that in general, the energy content of the radiative part of a system is not determined by the energy content of its matter part. Thus both energies should be taken into account as independent variables. In this section we consider a simple case by assuming the existence of near-equilibrium states such that the entropy of the matter (radiation) part depends only on the specific entropy of the matter (radiation) in the system, i.e.

$$s_{le} = s_m^{le}(u_m) + s_r^{le}(u_r) \quad (10)$$

(see also equation (6)). Here the sub- and superscripts  $le$  distinguish this case from the more general one in which the radiative energy flux  $\vec{F}$  may have to be taken into account as an additional variable (to be discussed in the next section). We define the matter and radiation temperatures,  $T_m$  and  $T_r$ , as

$$\frac{1}{T_m(u_m)} \equiv \frac{ds_m^{le}(u_m)}{du_m} = \frac{\partial s_{le}(u_m, u_r)}{\partial u_m} \quad (11)$$

† See [6], equations (22), (39) and (14).

and

$$\frac{1}{T_r(u_r)} \equiv \frac{ds_r^{le}(u_r)}{du_r} = \frac{\partial s_{le}(u_m, u_r)}{\partial u_r} \quad (12)$$

respectively. The last equalities in equations (11) and (12) follow directly from the assumption (10). We obtain a Gibbs-type equation for the system,

$$ds_{le} = \frac{1}{T_m} du_m + \frac{1}{T_r} du_r. \quad (13)$$

The question arises as to whether this formalism yields reasonable evolution equations or not. In order to derive them from equation (13), we need first to evaluate the rates of change of the specific energies.

We assume that the matter is macroscopically at rest (i.e., it has vanishing barycentric velocity). Then, the general form for the equation of balance of energy can be written as [2]

$$\rho \frac{du}{dt} = -\vec{\nabla} \cdot \vec{J}_u \quad (14)$$

where  $t$  is the time and  $\vec{J}_u$  is the total energy flux. In our case, we have a radiation–matter system (so that  $u$  is given by equation (1)) and energy is by assumption transported by radiation (so that  $\vec{J}_u = \vec{F}$ ), i.e.

$$\rho \frac{d}{dt}(u_m + u_r) = -\vec{\nabla} \cdot \vec{F}. \quad (15)$$

Equation (15) is an equation for the rate of change of the total specific energy, but it is clear from equation (13) that we need two such equations: one for the matter specific energy  $u_m$  and another one for  $u_r$  (radiative energy per unit mass of matter). In other words, the energy exchange between matter and radiation should be taken into account. The interaction between matter and radiation is described by the radiative transfer equation [10],

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \frac{\vec{c}}{c} \cdot \vec{\nabla} I_\nu = -\sigma I_\nu + e_\nu \quad (16)$$

where  $I_\nu$  is the radiation intensity, with  $\nu$  the frequency,  $\vec{c}$  is the photon velocity,  $c = |\vec{c}|$ ,  $\sigma$  is the absorption coefficient (here assumed independent of frequency for simplicity: the so-called grey approximation [10]) and  $e_\nu$  is the emission coefficient. The radiative energy density, namely  $\rho u_r$ , and the energy flux are related to the intensity according to [10]

$$\rho u_r = \frac{1}{c} \int_0^\infty d\nu \int_{4\pi} d\Omega I_\nu \quad (17)$$

$$\vec{F} = \int_0^\infty d\nu \int_{4\pi} d\Omega \frac{\vec{c}}{c} I_\nu \quad (18)$$

where  $d\Omega$  is a differential of solid angle. Integration of equation (16) over  $\nu$  and  $\Omega$  and use of equations (17) and (18) yields the equation for the rate of change of  $u_r$ ,

$$\rho \frac{du_r}{dt} = -\vec{\nabla} \cdot \vec{F} - \sigma c \rho u_r + 4\pi e_{tot} \quad (19)$$

where  $e_{tot} = \int_0^\infty d\nu e_\nu$  is the total emissivity. The equation for the rate of change of the specific matter energy can be obtained from equations (19) and (15),

$$\rho \frac{du_m}{dt} = \sigma c \rho u_r - 4\pi e_{tot}. \quad (20)$$

Use of equations (19) and (20) into (13) and rearrangement yields

$$\rho \frac{ds_{le}}{dt} = -\frac{\vec{\nabla} \cdot \vec{F}}{T_r} + (4\pi e_{tot} - \sigma c \rho u_r) \left( \frac{1}{T_r} - \frac{1}{T_m} \right). \quad (21)$$

Comparison of this result with the law of balance of entropy, namely [2]

$$\rho \frac{ds_{le}}{dt} = -\vec{\nabla} \cdot \vec{J}_{le}^s + \sigma_{le}^s$$

leads us to identify the entropy flux  $\vec{J}_{le}^s$  and entropy production rate  $\sigma_{le}^s$  as

$$\begin{aligned} \vec{J}_{le}^s &= \frac{\vec{F}}{T_r} \\ \sigma_{le}^s &= -\frac{\vec{F} \cdot \vec{\nabla} T_r}{T_r^2} + \frac{(4\pi e_{tot} - \sigma c \rho u_r)}{T_m T_r} (T_m - T_r). \end{aligned}$$

The second law requires that  $\sigma_{le}^s \geq 0$ . The simplest way to ensure this is to consider the following evolution equations:

$$\vec{F} = -\lambda_r \vec{\nabla} T_r \quad (\lambda \geq 0) \tag{22}$$

$$4\pi e_{tot} - \sigma c \rho u_r = \mu (T_m - T_r) \quad (\mu \geq 0). \tag{23}$$

Equation (22) is nothing but the radiative analogue to the Fourier heat conduction equation. On the other hand, equation (23) describes the energy equilibration between matter and radiation. To the best of our knowledge, this equation had not been explicitly obtained before. Use of equation (20) into the left-hand-side of (23) yields the usual equation for heat exchange between components of a mixture, in agreement with [11]. The first term on the left-hand side in equation (23) corresponds to the emission of radiation by matter, whereas the second one corresponds to the absorption of radiation. If  $T_m > T_r$ , according to equation (23) emission is more important than absorption, as was to be expected intuitively. Only if  $T_m = T_r$  there is compensation of emission and absorption; a common temperature can then be ascribed to matter and radiation.

#### 4. Systems out of local equilibrium

As stressed in the introduction, it is the purpose of this paper is to determine if a phenomenological model can be constructed such that it is consistent with the microscopic result (8). In order to tackle this problem, let us consider states more general than those in the previous section (see equation (10)), by now assuming that in general the radiative specific entropy  $s_r$  may also depend on the radiative heat flux  $\vec{F}$ :

$$s = s_m(u_m) + s_r(u_r, \vec{F}) \tag{24}$$

(we stress that such assumption has been confirmed from statistical mechanics, as explained above in equation (7)). The entropy differential reads

$$ds = \frac{1}{\theta_m} du_m + \frac{1}{\theta_r} du_r - \frac{\vec{\alpha}}{T_r \rho} \cdot d\vec{F} \tag{25}$$

where we have defined  $\theta_m$ ,  $\theta_r$  and  $\vec{\alpha}$  as

$$\frac{1}{\theta_m(u_m)} \equiv \frac{ds_m(u_m)}{du_m} = \frac{\partial s(u_m, u_r, \vec{F})}{\partial u_m} \tag{26}$$

$$\frac{1}{\theta_r(u_r, \vec{F})} \equiv \frac{\partial s_r(u_r, \vec{F})}{\partial u_r} = \frac{\partial s(u_m, u_r, \vec{F})}{\partial u_r} \tag{27}$$

$$\frac{\vec{\alpha}(u_r, \vec{F})}{T_r \rho} \equiv -\frac{\partial s_r(u_r, \vec{F})}{\partial \vec{F}} = -\frac{\partial s(u_m, u_r, \vec{F})}{\partial \vec{F}} \tag{28}$$

where the last equalities follow from equation (24). Comparison with equations (13), (11) and (12) shows that in situations where the dependence of  $s$  on  $\vec{F}$  is negligible, the last term in equation (25) no longer appears and we have  $\theta_m = T_m$  and  $\theta_r = T_r$ . Such cases have already been described in section 3. In order to generalize them but obtain relatively simple expressions, we consider states close enough to equilibrium so the last term in equation (25) can be approximated by its first-order MacLaurin expansion in  $\vec{F}$ . Then equation (25) becomes

$$ds = \frac{1}{\theta_m} du_m + \frac{1}{\theta_r} du_r - \left( \frac{\alpha(u_r)}{T_r \rho} \vec{F} + O(F^2) \right) \cdot d\vec{F} \quad (29)$$

where we have taken into account that for  $\vec{F} = 0$ , the generalized specific entropy  $s$  must reduce to the specific entropy  $s_{le}$  appearing in equation (13), so that the zeroth-order term in the expansion for  $\vec{\alpha}(u_r, \vec{F})$  vanishes, i.e. we can write  $\vec{\alpha}(u_r, \vec{F}) = \alpha(u_r) \vec{F} + O(F^2)$ , where  $O(F^2)$  stands for second- and higher-order terms in the components of  $\vec{F}$ . In order for equation (29) to be an exact differential, the following conditions must be fulfilled:

$$\begin{aligned} \frac{\partial^2 s(u_m, u_r, \vec{F})}{\partial \vec{F} \partial u_r} &= \frac{\partial^2 s(u_m, u_r, \vec{F})}{\partial u_r \partial \vec{F}} \\ \frac{\partial^2 s(u_m, u_r, \vec{F})}{\partial \vec{F} \partial u_m} &= \frac{\partial^2 s(u_m, u_r, \vec{F})}{\partial u_m \partial \vec{F}} \\ \frac{\partial^2 s(u_m, u_r, \vec{F})}{\partial u_m \partial u_r} &= \frac{\partial^2 s(u_m, u_r, \vec{F})}{\partial u_r \partial u_m}. \end{aligned}$$

We evaluate the first derivatives from equation (29), insert them into these three conditions and obtain that

$$\frac{1}{\theta_r} = \frac{1}{T_r} - \frac{\vec{F} \cdot \vec{F}}{2\rho} \frac{\partial}{\partial u_r} \left( \frac{\alpha(u_r)}{T_r} \right) + O(F^3) \quad (30)$$

$$\theta_m = T_m \quad (31)$$

$$\frac{\partial}{\partial u_m} \left( \frac{1}{\theta_r} \right) = \frac{\partial}{\partial u_r} \left( \frac{1}{\theta_m} \right). \quad (32)$$

In equations (31) and (32), use has been made of equations (11) and (12). By again taking them into account, it is easy to see that equations (30) and (31) imply that equation (32) is fulfilled. Use of equations (30) and (31), as well as of the equations of balance of energy (19) and (20), into equation (29) yields the following evolution equation for the entropy density:

$$\rho \frac{ds}{dt} = -\frac{\vec{\nabla} \cdot \vec{F}}{\theta_r} + (4\pi e_{tot} - \sigma c \rho u_r) \left( \frac{1}{\theta_r} - \frac{1}{\theta_m} \right) - \left( \frac{\alpha(u_r)}{T_r \rho} \vec{F} + O(F^2) \right) \cdot \frac{d\vec{F}}{dt} \quad (33)$$

which generalizes equation (21). We can now proceed in the usual way in extended irreversible thermodynamics [4] and identify, from the law of balance of entropy [2, 4]

$$\rho \frac{ds}{dt} = -\vec{\nabla} \cdot \vec{J}^s + \sigma^s$$

the entropy flux  $\vec{J}^s$  and entropy production rate  $\sigma^s$  as

$$\begin{aligned} \vec{J}^s &= \frac{\vec{F}}{\theta_r} \\ \sigma^s &= -\frac{\vec{F}}{\theta_r^2} \cdot \left( \vec{\nabla} \theta_r + [\alpha(u_r) T_r + O(F)] \frac{d\vec{F}}{dt} \right) + \frac{(4\pi e_{tot} - \sigma c \rho u_r)}{\theta_m \theta_r} (\theta_m - \theta_r) \end{aligned}$$

where use has been made of equation (30). The simplest way to ensure the second law, namely  $\sigma^s \geq 0$ , is to consider the following evolution equations:

$$\vec{F} = -\lambda \left( \vec{\nabla}\theta_r + [\alpha(u_r)T_r + O(F)] \frac{d\vec{F}}{dt} \right) \quad (\lambda \geq 0) \quad (34)$$

$$4\pi e_{tot} - \sigma c \rho u_r = \mu(\theta_m - \theta_r) \quad (\mu \geq 0). \quad (35)$$

These equations generalize equations (22) and (23) to states further-away from equilibrium, implying that in such states the generalized temperatures  $\theta_r$  and  $\theta_m$  drive the energy transfer processes. Such a result is not surprising from the perspective that the definitions (26) and (27) are rather natural nonequilibrium extensions of the (local-)equilibrium definition of temperature (2), which was used by Boltzmann in order to derive the Stefan law, in complete agreement to experiment [12]. In our case we have  $\theta_m = T_m$  (see equation (31)) because we have assumed that heat is transported solely by radiation; it is not difficult to include, e.g., a conductive heat flux and obtain that  $\theta_m \neq T_m$ ; however, here we are interested in radiative transfer. Equation (34) can be written in the form

$$[\tau_r + O(F)] \frac{d\vec{F}}{dt} + \vec{F} = -\lambda_r \vec{\nabla}\theta_r \quad (36)$$

with  $\tau_r$  given by

$$\alpha(u_r) = \frac{\tau_r}{\lambda_r T_r}.$$

Finally, we rewrite equations (30) and (29) in terms of  $\tau_r$ ,

$$\frac{1}{\theta_r} = \frac{1}{T_r} - \frac{\vec{F} \cdot \vec{F}}{2\rho} \frac{\partial}{\partial u_r} \left( \frac{\tau_r}{\lambda_r T_r^2} \right) + O(F^3)$$

$$ds = \frac{1}{\theta_m} du_m + \frac{1}{\theta_r} du_r - \frac{\tau_r}{\lambda_r T_r^2 \rho} \vec{F} \cdot d\vec{F}.$$

## 5. Single-temperature systems: comparison with statistical mechanics

We can now consider the special case in which energy equilibration has taken place. As explained in the introduction, this corresponds to the steady state attained by the system after a sufficiently long time and under a constant incident radiation field. In such an instance, matter and radiation have equilibrated their temperatures (i.e.,  $\theta_m = \theta_r$ , see equation (35); note that equation (19) yields  $\vec{\nabla} \cdot \vec{F} = 0$ ) and only then we may introduce an overall temperature  $\theta$  for the system, i.e.

$$\frac{1}{\theta} \equiv \frac{1}{\theta_m} = \frac{1}{\theta_r} = \frac{1}{T_r} - \frac{\vec{F} \cdot \vec{F}}{2\rho} \frac{\partial}{\partial u_r} \left( \frac{\tau_r}{\lambda_r T_r^2} \right) + O(F^3) \quad (37)$$

$$ds = \frac{1}{\theta} du - \frac{\tau_r}{\lambda_r T_r^2 \rho} \vec{F} \cdot d\vec{F} \quad (38)$$

where we have applied equation (1). Finally, equation (36) becomes

$$\tau_r \frac{d\vec{F}}{dt} + \vec{F} = -\lambda_r \vec{\nabla}\theta. \quad (39)$$

Equation (37) (which has been derived here phenomenologically) is exactly the same as equation (8) (which is the result obtained from nonequilibrium statistical mechanics, as already stressed above). This result is also in agreement with earlier work based on fluctuation theory [13]. Equation (37) may be used in order to estimate  $\theta$  in radiative systems, e.g. stellar surfaces

and interiors (although in this specific case, the values of the variables  $\vec{F}$ ,  $\rho$  and  $T_r$  cannot be directly measured, and the generalization of the existing stellar models within the framework of extended thermodynamics has not yet been developed [1]).

There is also complete agreement between equation (38) and the corresponding statistical-mechanical result<sup>†</sup>. For the sake of completeness, in the appendix we also check the consistency of the radiative heat transport equation (39), which has been derived here from thermodynamics, with the results from the microscopic theory.

## 6. Concluding remarks

By taking into account that the thermal state of matter in an arbitrary physical system may be different from that of radiation in the same system, we have been able to present a thermodynamical model that takes into account the energy equilibration between matter and radiation. This is described by equation (23) in near-equilibrium systems, and its generalization (35) in system further away from equilibrium.

Our model yields (as a special case) a phenomenological model that, in contrast to a less elaborate one developed previously [1], is in perfect agreement with more recent results based on statistical mechanics of radiation–matter systems in which energy exchange between matter and radiation is negligible. The discrepancy is solved when one reaches the result, derived here, that in general nonequilibrium states matter and radiation do not share their respective local-equilibrium temperatures, i.e.,  $T_m \neq T_r$  even in the case that  $\theta_m = \theta_r$  (see equations (31) and (37)). We would like to stress that (i) matter–radiation energy equilibration is also included in this paper; and (ii) the model presented here is of a purely thermodynamical nature and much more general than the microscopic approaches in [5, 6, 8, 9], because those approaches did not allow for matter–radiation equilibration processes and were restricted to certain assumptions on the matter content of the system.

We now have a reasonable, fully consistent thermodynamical picture, which is in agreement with all of the statistical-mechanical results so far published. In closing we mention that the methods in this paper may also be of interest in topics other than nonequilibrium radiation, particularly in the analysis of energy relaxation in matter systems composed of several components with different temperatures, typical cases being found in plasma physics [14], chemically-reacting systems [15] and short-pulse laser heating of metals [16].

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## Appendix A. Microscopic derivation of the heat transport equation

In this appendix we derive the heat transport equation from previous information statistical results and show that it is consistent with the corresponding equation derived here from thermodynamics in the case that matter and radiation share a single temperature, namely equation (39).

Within information-statistical theory, the near-equilibrium radiation momentum distribution function is expanded about the equilibrium (or Planckian) distribution  $f_r^{(0)}$ ,

$$f_r = f_r^{(0)}(1 + \phi^{(1)} + \phi^{(2)} + O(F^3))$$

<sup>†</sup> See [6], equations (23), (39) and (14).

where  $\phi^{(i)}$  may contain terms proportional to the  $i$ th-power of the components of  $\vec{F}$ . The terms in this expansion can be written either in terms of  $T_r$  or in terms of  $\theta = T_m$  (see equations (31) and (37)). The former approach was used in, e.g., [5], whereas the latter one was used in [6]. The results of both approaches are seen to be consistent, as they should, provided that one keeps always in mind the order up to which the analysis is being performed.

We begin by noting that the phenomenological approach in this paper corresponds to taking into account nonequilibrium corrections to the entropy up to second order (see equation (29)), i.e. an specific entropy of the form

$$s = s^{(0)} + s^{(1)} + s^{(2)} + O(F^3)$$

where  $s^{(i)}$  may contain terms proportional to the  $i$ th-power of the components of  $\vec{F}$ . We have shown previously, from the information-statistical expansion for  $f_r$ , that  $s^{(1)}$  vanishes identically and that  $\phi^{(2)}$  does not contribute to  $s^{(2)\dagger}$ . It means that the thermodynamical analysis in this paper corresponds to the first-order approximation to  $f_r$ , namely

$$f_r = f_r^{(0)}(1 + \phi^{(1)} + O(F^2)) \tag{A1}$$

where $\ddagger$

$$f_r^{(0)} = \frac{1}{e^{p_r c/k\theta} - 1} \tag{A2}$$

$$\phi^{(1)} = -\frac{3p_r}{4ak\theta^5} \frac{e^{p_r c/k\theta}}{e^{p_r c/k\theta} - 1} \vec{F} \cdot \frac{\vec{c}}{c} \tag{A3}$$

with  $\vec{c}$  the photon velocity and  $a = \frac{8\pi^5 k^4}{15c^3 h^3}$  the blackbody constant. On the other hand, multiplication of equation (16) by  $\frac{\vec{c}}{c}$ , integration and use of equation (18) yields

$$\frac{1}{c} \frac{d\vec{F}}{dt} + \sigma \vec{F} = -2 \sum_{k=1}^3 \frac{\partial}{\partial x_k} \int \frac{d^3 p_r}{h^3} p_r \vec{c}_k f_r$$

where we have applied that  $d^3 p_r = p_r^2 d\Omega = h^3 v^2 dv d\Omega/c^3$  and  $I_v = 2hv^3 f_r/c^2 = 2p_r^3 c f_r/h^2$  [17]. It is simple to substitute equations (A1)–(A3) and integrate. In this way, one finally finds that

$$\frac{1}{c} \frac{d\vec{F}}{dt} + \sigma \vec{F} = -\frac{4ca\theta}{3} \vec{\nabla}\theta$$

and this is exactly equation (39). Whereas the microscopic approach allows us to identify  $\tau_r = 1/c\sigma$  and  $\lambda_r = 4ca\theta^3/3\sigma$ , it should be kept in mind that such an approach was derived under certain assumptions for the matter content of the system [5, 8]. In contrast, the theory presented in this paper is general, mathematically simpler, and also provides an adequate framework to take into account the matter–radiation equilibration of energy (see equation (35)).

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$\dagger$  See [6], p 351.

$\ddagger$  See [5], equation (33). Note from equation (A.14) in that reference and the text below it, that the notation is different:  $T$  in [5] corresponds to  $\theta$  in this paper.

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