

No part of this digital document may be reproduced, stored in a retrieval system or transmitted in any form or by any means. The publisher has taken reasonable care in the preparation of this digital document, but makes no expressed or implied warranty of any kind and assumes no responsibility for any errors or omissions. No liability is assumed for incidental or consequential damages in connection with or arising out of information contained herein. This digital document is sold with the clear understanding that the publisher is not engaged in rendering legal, medical or any other professional services.

*Chapter 5*

## ON MOTT CONDUCTIVITY EXPONENTS OF PSEUDO-GAP AMORPHOUS SYSTEMS

*Debnarayan Jana*<sup>1\*</sup> and *Joaquim Fort*<sup>2†</sup>

<sup>1</sup>Department of Physics, University College of Science and Technology  
92 A. P. C. Road, Kolkata -700 009, W.B., India

<sup>2</sup>Departament de Física, Universitat de Girona, 17071 Girona, Spain

### Abstract

Mott first pointed out the importance of electron's Variable Range Hopping (VRH) between the localized states in discussing the conduction process of disordered insulators at low temperature. In this article, we use VRH Model to compute the Mott conductivity of an interacting amorphous system in an arbitrary dimension  $d$ . The characteristic feature of this interacting system is the existence of a soft gap at the Fermi energy. We found that both at low as well as at high electric field, the values of the exponents characterizing the behavior of conductivity in this interacting system are always higher than that of non-interacting ones in all spatial dimensions. We also obtain the bound (lower as well as upper) of the values of exponents in contrast to non-interacting case. Finally, this approach allows us to construct a simple form for A.C. conductivity as a function of frequency. We also indicate a general form of the conductivity as a function of electric field and temperature for this system in any arbitrary dimension  $d$ . The scaling approach adopted here to study the Mott conductivity gives one the generalized forms of the exponents from which all the previously known results can be obtained. All these may shed light to interpret the experimental conductivity results of amorphous systems at low temperature.

**Keywords:** Mott Conductivity, Localized State, Extended State, Pseudo Gap, Amorphous Material.

---

\*E-mail address: d.jana@cucc.ernet.in. Visiting Researcher at Center for Condensed Matter Sciences, National Taiwan University, Taipei, Taiwan

†E-mail address: joaquim.fort@udg.es

## 1 Introduction

Regular crystalline solid is characterized by extended Bloch wave function. This description of a perfect crystalline solid is possible because of the presence of strong *translational* symmetry in the system. In presence of a structural disorder, however, this symmetry is no longer satisfied and the electronic wave functions are no longer of the Bloch type anymore. The wave function in such a situation becomes spatially localized just like a wave packet with a finite extent. This peculiar nature of the disordered system was independently shown by P. W. Anderson[1] and N. F. Mott[2]. Moreover, in contrast to these crystalline materials, amorphous materials have a continuous density of states in a range of energies where the states are localized. The existence of these localized states or traps has been experimentally verified by measuring the transit time of injected charge carriers[3].

By definition, the localized states do not carry any current in the thermodynamic limit. In other words, the configurational (or ensemble) average of the conductivity for wave functions having a fixed energy must vanish ( $\langle \sigma_E \rangle = 0$ ) at absolute zero temperature as the volume of the system is tending to infinity. The localization degree of such a state is characterized by a length known as localization length which measures the spatial extension of the wave function of electron. Thus, the conduction involving the localized states can only take place through the transitions of electrons from full states to neighboring empty states with the help of phonons. On the other hand, the extended states which can carry current at  $T = 0$  (i. e. finite conductivity) can be distinguished from the localized one by a quantity known as Generalized Inverse Participation Ratio (GIPR). The GIPR is defined as

$$P^r(q, L) = \left\langle \frac{\sum_i |\psi_i^r|^{2q}}{(\sum_i |\psi_i|^2)^q} \right\rangle \quad (1)$$

and can be regarded as the  $q$  th moment of  $i$ -th element of  $r$ -th eigenstate and  $q$  is real positive number greater than 1. The angular bracket  $\langle \dots \rangle$  denotes the disorder averaging of the relevant quantity and  $L$  is the typical system size. It was Thouless[4] who first introduced a physical interpretation of IPR ( $q = 2$ ), a subset of GIPR in disordered systems through Green's function technique. GIPR has been mostly used in numerical studies of metal-insulator transition[5, 6] and continuous disordered model in a magnetic field[7, 8]. It is found from numerical calculations that  $P^r(q, L) \sim \frac{1}{L^{\tau(q)}}$ . In the continuum version, a straightforward calculation in case of pure delocalized states ( $\exp(i\vec{k} \cdot \vec{r})$ ) in any arbitrary  $d$  dimension reveals that  $\tau(q) = d(q - 1)$ . Thus, in the thermodynamic limit, for extended states GIPR vanishes to zero. While for the extremely localized state ( $\exp(-\alpha r)$ ), in any arbitrary dimension  $d$  GIPR is non-zero in the thermodynamic limit and is given by

$$GIPR \sim \frac{(2\alpha)^{d(q-1)}}{q^d} \quad (2)$$

For a given eigenstate with fixed system size, a higher value of  $\tau(q)$  for extended state indicates that GIPR is smaller for a delocalized state than a localized state. In any intermediate system size,  $\tau(q)$  is a *linear* function of  $q$  while it is a *non-linear* function of  $q$  for localized one. This non-linear nature of  $\tau(q)$  indicates the typical multi-fractal character of localized state. This GIPR has been recently used in two-dimensional (tight-binding

model) disordered (on site disorder only) system in a magnetic field to explore the nature of the energy eigen states within a band[9]. In this discrete model, the effect of the magnetic field was introduced in the lowest order through the phase of the hopping integral between two sites as suggested by Peierls[10]. This approach has been extended to include also the random flux model with on site disorder in a two-dimensional system.

Recently, amorphous materials have attracted considerable attention with regard to their optical as well as electrical properties[3, 11, 12, 13]. To understand the behavior of their electrical conduction, Mott[14] proposed a new mechanism of transport of electrons long back ago. This mechanism is effective only at sufficiently low temperature and is known in literature as Variable Range Hopping (VRH). This VRH model has been applied in wide varieties of systems in condensed matter physics. In recent years, VRH model has been extensively used in metal insulator transition[15, 16, 17], insulating amorphous alloys[18], thin film transistors[19], n type CdSe semiconductor nanocrystal solids[20], inorganic compounds[21] and in mesoscopic carbon networks[22]. A simple variant of this VRH model known as quasi-1D VRH model[23] has been invoked to consider the charge transport in the disordered regime of HCl doped PAN-ES samples. The crossover phenomena in conjugated polymer[24] has also been studied recently via this model. VRH has also been used in studying the conductivity of La-based CuO-chained high temperature superconductors[25]. This immediately points out the importance of VRH model in condensed matter physics. Hence, a proper understanding of this simple model is necessary in order to interpret the low temperature conductivity results. In this article, we would like to discuss the basic aspects of this model and its natural extension to an interacting system.

This chapter is organized as follows. In the next section, we give a brief introduction to the VRH model and then in section 3, we use this model to compute the d.c. conductivity of pseudo-gap amorphous materials. Section 4 is devoted to discuss the bound on the values of the exponents in this interacting case. Then, we move on to discuss the scaling behavior of the the A.C. conductivity as a function of frequency. In section 6, we take up two experimental systems discussed recently in the literature to show the applicability of VRH model. Finally, we give our conclusion in section 7.

## 2 The Variable Range Hopping Model

The atoms in amorphous material are distributed at random and the electrons associated with them have distribution of energies. An atom might have an empty state whose energy is slightly higher than that of the occupied state of a chosen given atom. Typically, the activation energy required for electrons to hop to an empty state is very small and hence the hopping conduction[3] takes place. The hopping distance varies because of the random arrangement of the atoms having suitable energy scales and hence the name is given as Variable Range Hopping (VRH). This hopping process differs from the usual electrical conduction in normal metal. Here, in the hopping process, phonons assist the transport while in typical band theory, the transport is impeded by the destruction in periodicity caused by the lattice vibration.

We consider two localized states - one filled and at or slightly below the Fermi energy  $E_F$  and the other empty above  $E_F$ ; their energy and spatial separations are  $W$  and  $R$

respectively. The hopping transition rate  $p$  is given by

$$p = \nu_{ph} \exp(-2\alpha R - \beta W(R)) \quad (3)$$

The first factor is the quantum mechanical tunnelling probability and is just the overlap of the two localized states ( one below the Fermi energy  $E_F$  and the other above  $E_F$ ) decaying with the same characteristic localization length  $\alpha^{-1}$ .  $R$  is the typical hopping distance between the localized states,  $\beta = \frac{1}{k_B T}$  and  $W$  is the energy separation between the final and initial state. The second factor arises because of the fact that the different localized states must have different energy. This difference in energy  $W(R)$  is supported by the phonon scattering and governed by the Boltzmann weight at temperature  $T$ . Note that  $W(R)$  essentially depends on separation between the localized states. At high enough temperature or high electric field, the variable range hopping distance  $R$  becomes equal to the nearest neighbor distance. The attempt frequency  $\nu_{ph}$  depends on the strength of the electron-phonon coupling and the phonon density of states but almost independent of  $R$  and  $W$ . This simple picture of hopping however can be rigorously justified by percolation theory[27, 28] and random resistor network model[29, 30].

In presence of a uniform electric field, however, we note that the conductivity will be modified[3] by the term due to electric field as

$$\sigma(F) \sim \exp(\pm eFR/k_B T) \quad (4)$$

Now, adding these two probabilities, we get the conductivity as

$$\sigma \sim \nu_{ph} \exp(-2\alpha R - \beta W) \sinh\left(\frac{eFR}{k_B T}\right) \quad (5)$$

Under weak field limit ( $eFR \ll k_B T$ ), the current density can be written as

$$j \sim 2e^2 F R^2 N(E_F) \nu_{ph} \exp(-2\alpha R - \beta W) \quad (6)$$

This immediately implies that the conductivity is given by

$$\sigma = 2e^2 R^2 N(E_F) \nu_{ph} \exp(-2\alpha R - \beta W) \quad (7)$$

An electron will always try to have lowest  $W$ , the energy difference between the localized states as well as the lowest hopping distance  $R$ . However, these two conditions cannot be satisfied simultaneously. In fact, the electrons have a tendency to hop to more distant neighbors where the energy difference  $W$  is smaller. This automatically gives rise to an optimum value of the hopping distance  $R$  obtained by maximizing the transition probability. This maximization is simply done by the optimization of the exponent given by  $\mathcal{P} = 2\alpha R + \beta W$ , where  $W$  depends on  $R$ [14].

For a constant density of states in 3 dimensions (subjected to the constraints that the energy window  $k_B T$  is quite narrow and the localization length  $\alpha^{-1}$  and the Fermi energy  $E_F$  remain constant within the narrow energy window) Mott[14] found that the conductivity which is related to the transition probability varies with temperature as

$$\sigma(T) = \sigma_0 \exp\left(-\frac{A}{T^{1/4}}\right). \quad (8)$$

This non-exponential rather stretched exponential behavior of the conductivity is one of the main features of the VRH model. We also note that with decrease (increase) of temperature the conductivity decreases (increases). The exponent  $1/4$  has been observed in many experiments[3]. Even for multi particle hopping instead of the above single particle one, Knotek and Pollak[31] obtained the similar behavior with exponent greater than  $\frac{1}{4}$  in three dimensions. Even in dilute interacting model, the exponent  $\frac{1}{2}$  was obtained in 3 dimensions[32, 33]. This change of exponent from  $\frac{1}{4}$  to  $\frac{1}{2}$  was regarded as tuning the Coulomb interaction over the non-interaction. There is a simple physical way[12] to understand this power  $\frac{1}{4}$ . In real 3 dimensional world, an electron will hop to a particular site in four  $(x, y, z, E)$  dimensional phase space. Therefore, this factor  $\frac{1}{4}$  can be regarded as the reciprocal of the effective dimensionality of 4 ( 3 spatial and one energy) of the electron's hopping phase space. In an arbitrary spatial dimension  $d$ , a straightforward generalization[12] gives the value of the exponent as  $\frac{1}{d+1}$ . Without doing much calculation, we can immediately obtain the exponent  $\frac{1}{3}$  for a typical two dimensional non-interacting electron's hopping. There are considerable amount of discussion in the literature regarding the value of the exponent[34, 35]. We Would like to generalize the above formula for an interacting system in which the density of states has a soft gap at the Fermi surface. This soft gap is obtained when one considers the electron-electron Coulomb interaction. Undoubtedly, this is more realistic than a constant density of states[14, 36, 37]. This is why it is interesting as well as relevant to analyze its effect on the conductivity exponent in the light of varying density of states in an arbitrary spatial dimension  $d$ . In other words, we want to look for how does the conductivity exponent vary with the strength of the density of states in such a typical interacting amorphous system. There are evidences that the experimental data lead to different exponents for the conductivity within a temperature window[38]. To explore the origin of these exponents, we would like to investigate this old classic problem[39]. Recently, a unifying theory of dc transport[40] for two dimensional interacting electronic crystals have been developed in the literature. In this paper[40], it has been shown that a crossover from a usual quadratic Coulomb gap to another one having density of states proportional to  $E^{2/3}$  leads to a change of conductivity exponent from  $\frac{3}{5}$  to  $\frac{5}{11}$ . Within our simple scaling approach, it is however possible to obtain the values of the exponents. Moreover, because of the bound of the pseudo gap exponents, we do have a lower bound of the conductivity exponents in all spatial dimensions as presented below.

### 3 Computation of D.C. Conductivity

In this section, we would like to generalize the non-interacting result to interaction one following the single-particle DOS as

$$N(E) \sim |E - E_F|^\nu. \quad (9)$$

The non-negative parameter  $\nu$  actually determines the way the DOS vanishes at the Fermi surface. This particular nature of the density of states arises while studying the localized electrons interacting via Coulomb interaction[32] at low temperature. A bound for the exponent  $\nu$  was also obtained as being[41]

$$\nu \geq d - 1 \quad (10)$$

for an arbitrary dimension  $d$ . With the above density of states, the unitarity condition within the specified energy intervals  $E_F \pm W(R)$  in an arbitrary dimension  $d$  reads as

$$\frac{2\pi^{d/2}R^d}{\Gamma(d/2 + 1)} \frac{W^{\nu+1}}{(\nu + 1)} = 1. \quad (11)$$

It is clear from the above equation (11) that  $W(R)$  is essentially the average energy spacing among states spatially located within a hyper sphere of radius  $R$ , and thus we note that

$$W(R) \sim \frac{1}{R^{d/(\nu+1)}}. \quad (12)$$

For an alternative derivation using the average no. of states, the readers are referred to the literature[39]. Incorporating the above density of states in Fermi-Dirac distribution function, the equation (7) takes the form

$$\sigma \sim 2e^2 R^2 T^\nu \nu_{ph} \exp(-2\alpha R - \beta W) \quad (13)$$

It is also evident that for  $\nu = 0$  (flat DOS)  $W(R)$  is proportional to  $R^{-d}$ . Hence, the exponent  $\mathcal{P}$  in the hopping transition rate in equation (1),  $p = \nu_{ph} \exp(-\mathcal{P})$  becomes

$$\mathcal{P} = 2\alpha R + \frac{\beta b}{R^{d/(\nu+1)}}. \quad (14)$$

Note that  $b$  is a constant independent of  $R$ ,  $W$  and  $T$ . Maximizing the hopping transition rate with respect to  $R$  we obtain

$$\sigma(T) = \sigma_0 T^{B(\nu,d)} \exp\left(-\frac{A}{T^\phi}\right), \quad (15)$$

where the exponent  $\phi$  is given by

$$\phi_{int}(d, \nu) = \frac{\nu + 1}{d + \nu + 1}, \quad B(\nu, d) = \frac{\nu^2 + d\nu - \nu - 2}{d + \nu + 1} \quad (16)$$

which matches with an earlier result[42]. Even for a power law type of density of states ( $N(E) \propto E^\nu$ ), the same exponent  $\phi(d, \nu)$  was obtained recently[43]. This equation (16) will be used later on to consider the conductivity of this interacting system in a high field. Before we go on next section, we would like to comment on two important aspects of the exponents. First of all, is there any simple physical way to understand these exponents? A brief look into equation (12) however answers this question. Interaction restricts the spatial part of effective phase space dimension of electrons to  $\frac{d}{\nu+1}$  making the total dimensions to  $\left(\frac{d}{\nu+1} + 1\right)$ . Then, its reciprocal readily gives the exponent  $\phi_{int}(d, \nu)$ . Secondly, it is noticed that the exponents in the interacting case in *all dimensions*  $d$  satisfy the following inequality given by

$$\phi_{int} > \phi_{non} \quad (17)$$

where  $\phi_{non}$  is the corresponding exponents for non-interacting case. In other words, the magnitude of the conductivity is increased in compared to non-interacting case. In figure 1, the behavior of the conductivity of the interacting in 3d is contrasted with the non-interacting along with DOS.

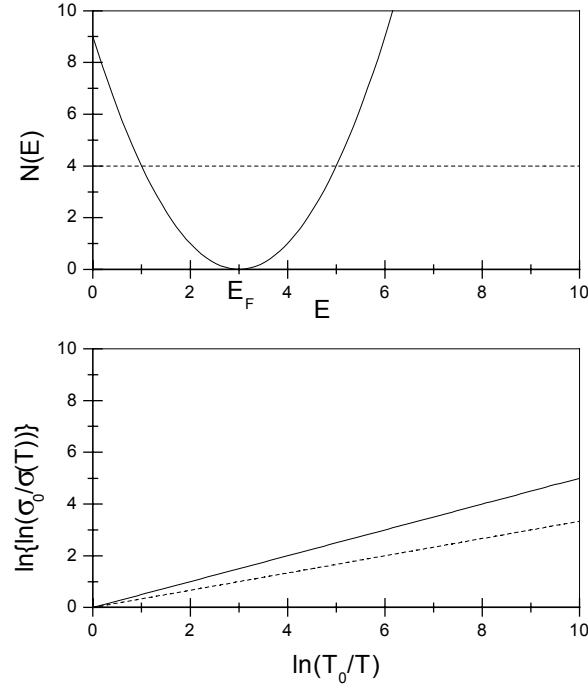


Figure 1: Figure shows the schematic behavior of DOS and (d.c.) conductivity in the interacting case in compared to non-interacting one in 3d. In both plots, the solid line is for interacting system while dashed one for non-interacting system.

A simple physical argument can be given to support this behavior of the exponents. It is evident that in the interacting case ( $\nu \neq 0$ ), the average no. of states increases. This results a slower power fall of the separation of the energy between the localized states. This, in turn, enhances the effective total hopping space dimensionality of the electrons, resulting an increase in the conductivity.

### 3.1 Extension to High Electric Field

The result discussed above is strictly valid for low temperatures and weak electric fields only. In extremely weak electric field ( $F$ ), the electric field-dependent conductivity[3] is given by

$$\sigma(F) \sim \exp(eRF/k_B T) \quad (18)$$

where  $R$  is the most probable hopping distance.

Experimentally, this low-field conductivity dependence on electric field and temperature has been observed[44] in P doped Si samples. We have recently [39] found the conductivity form of this system at high electric field as

$$\sigma(T) = \sigma^{HF} \exp\left(-\frac{B}{F\phi}\right), \quad \phi_2 = \frac{\nu + 1}{d + \nu + 1}. \quad (19)$$

Therefore, like non-interacting case, we note that at sufficiently high electric field intensities, there is no temperature dependence of the conductivity. An intuitive argument in

support of such a dependence on the field can be put forward as follows. We here generalize the reasoning originally given for non-interacting system[3] in three spatial dimensions. At strong electric field, the energy associated with the field  $F$  is much greater than the hopping energy available. This immediately restricts the electron's hopping over localized sites to downwards only. Hence, for the occurrence of this process, we must have a critical field  $F_c$  defined by

$$F \geq \frac{C_1}{R^{\frac{d+\nu+1}{\nu+1}}} \quad (20)$$

Thus, for  $F > F_c$ , the hopping distance  $R$  naturally should scale as  $F^{-\frac{\nu+1}{d+\nu+1}}$ . Finally, one can easily obtain the equation (19) by simply considering  $\sigma \sim \exp(-2\alpha R)$  and substituting the scaling form of  $R$ . Previously-known results[45, 46, 47] can be obtained from this generalized form of the conductivity in high electric fields. In the special case of three dimensional systems and a quadratic DOS ( $\nu = 2$ ), the familiar results of refs. [42, 47] can be recovered easily.

A combined formula for any electric field  $F$  and temperature  $T$  in an arbitrary dimension  $d$  can be proposed using the concept developed in case of coulomb gap in three dimension[20]. Using this concept of partition function to normalize the distribution in statistical mechanics, we find that the conductivity apart from the pre-exponential term should behave as

$$\sigma \sim \frac{\exp(-2\alpha R - \beta W - \beta eFR)}{1 + \exp(-\beta W - \beta eFR)} \quad (21)$$

It is easy to verify that at low enough temperature and low electric field the above equation (21) correctly reduces to (15). At high electric field, there is no temperature dependence, it goes back simply to field dependence equation given by (19). Physically speaking, at low electric field the electrons hop in such a way as to reduce the activation energy  $W$  while at high electric field, they overcome this energy  $W$  with the help of energy from the field. In fact, in these two regimes, the hopping distance should behave differently as a function of field and temperature. To understand this more clearly, we look for the variation of hopping distance to maximize the above conductivity expression (21). This enables us to get an equation for  $R$  as

$$\begin{aligned} & \left\{ \frac{d}{\nu+1} \frac{\beta S_d}{R^{\frac{d+\nu+1}{\nu+1}}} - (\beta eF + 2\alpha) \right\} \left( 1 + \exp \left\{ -\frac{\beta S_d}{R^{\frac{d}{\nu+1}}} - \beta eFR \right\} \right) \\ &= \left( \frac{d}{\nu+1} \frac{\beta S_d}{R^{\frac{d+\nu+1}{\nu+1}}} - \beta eF \right) \exp \left\{ -\frac{\beta S_d}{R^{\frac{d}{\nu+1}}} - \beta eFR \right\} \end{aligned} \quad (22)$$

A simplified equation of  $R$  in  $d = 3$  and  $\nu = 2$  (Coulomb Gap) valid for any value of  $F$  and temperature  $T$  can be written simply as

$$\left\{ \frac{\beta S_3}{R^2} - (\beta eF + 2\alpha) \right\} \left( 1 + e^{-\frac{\beta S_3}{R} - \beta eFR} \right) = \left( \frac{\beta S_3}{R^2} - \beta eF \right) e^{-\frac{\beta S_3}{R^2} - \beta eFR} \quad (23)$$

Here,  $S_d(S_3)$  is a dimensionless constant independent of  $R$ ,  $W$ ,  $F$  and  $T$ . From this equation (23), we find for low electric field  $R \sim \beta^{1/2}$  [32] while for high electric field  $R \sim F^{-1/2}$ . This immediately points out that why in low electric field, the conductivity depends on the temperature but at high field, the conductivity is independent of temperature.



It is also noticed here that in *all spatial dimensions*  $d$

$$\phi_2 > \phi_0 \quad (24)$$

where  $\phi_0$  is the exponents for the non-interacting case ( $\nu = 0$ ). The physical arguments leading to this behavior is the same outlined above for low electric field and temperature i. e. the enhancement of the phase space dimensionality in the interacting case. Below we discuss the lower as well as the upper bounds of the exponents.

## 4 Bounds on the Exponents

In this section we would like to discuss the limiting values of the exponents of the interacting case. The exponent  $B(\nu, d)$  in the pre-exponential factor of the Mott conductivity formula deserves some discussion. In most of the experimental situation, however, the variation is insignificant. There could be some possible situations[48, 49] where the temperature factor in the pre-exponential term might play a crucial role to fit the experimental data. Therefore, we suggest to take into account this temperature factor in the pre-exponential term[50] appropriately for scaling fit of experimental conductivity data according to Variable Range Hopping (VRH) model. It is interesting to note that the exponent satisfies the following inequality[51] given by

$$(d - 2) \leq B(\nu, d) < \nu \quad (25)$$

For comparison, we note that in non-interacting case, the value of  $B(\nu = 0, d) = -\frac{2}{d+1} \leq -1$ . It is evident from the expressions (16) and (19) that the exponents  $\phi_{int}$  or  $\phi_2$  always have the upper limit as 1. Thus, we have

$$\phi_{int} < 1 \quad (26)$$

Now considering the bound on the values of  $\nu$  given by equation (10), we find a new lower limit of the exponents given by

$$\phi_{int} \geq \frac{1}{2} \quad (27)$$

Therefore, for this special type of interacting system we have the bounds on the exponents as

$$\frac{1}{2} \leq \phi_{int} < 1, \quad \frac{1}{2} \leq \phi_2 < 1 \quad (28)$$

Notice that this lower bound is possible only because of the restriction of the exponent  $\nu$ . There is no such kind of lower bound for non-interacting system ( the values are less than  $\frac{1}{2}$  in two and three dimensions) as the values are discrete, not functions of  $\nu$ . In non-interacting system in one dimension however the value of  $1/2$  is possible; but there is no such counterpart in higher dimensions. Except in one dimension, the exponents do strictly obey the inequality given above. In non-interacting cases, except for one dimension, all the exponents have values less than  $\frac{1}{2}$ . Therefore, we can conclude that any value greater than  $\frac{1}{2}$  is a characteristic feature of this interacting system having pseudo-gap at the Fermi surface. In figure 2, we show the variation of the exponent of the interacting system in two as well as three dimension as a function of  $\nu$ . This result is also valid for the high field case because of the same exponent dependence on the field as in the low temperature conductivity case.

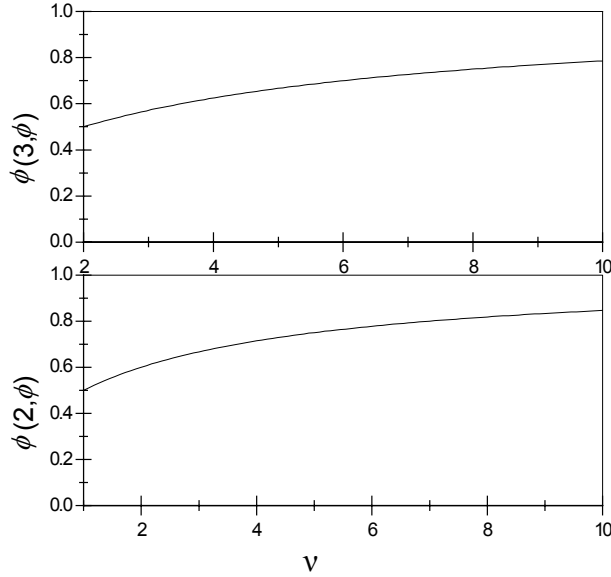


Figure 2: In this figure, the variation of the exponent  $\phi$  for the interacting cases in three and two dimensions as a function of  $\nu$  is indicated.

## 5 Calculation of A.C. Conductivity

In this section we closely follow Austin-Mott[52] formulation to compute the scaling behavior of the A.C. conductivity. The scaling behavior in the non-interacting limit[53] suggests the form of this A.C. conductivity in three dimension as

$$\sigma(\omega) \sim T\omega \ln\left(\frac{\nu_{ph}}{\omega}\right)^4 \quad (29)$$

The significant role played by  $\nu_{ph}$  is discussed below. It is known[53] that for flat density of states,

$$\sigma_{ac} = \sigma_{dc} + A_1\omega^s, \quad \omega \ll \nu_{ph} \quad (30)$$

Here,  $A_1$  is a function of temperature and  $s$  is a weak function of the frequency  $\omega$ . In three dimensions, the value of  $s$  under the above limiting conditions is 0.8. Based on the above definition, one may define  $s$  in the following way

$$s = \frac{d[\ln\{\omega \ln^\alpha(\nu_{ph}/\omega)\}]}{d(\ln \omega)} \quad (31)$$

where it is assumed that the behavior of the A.C. conductivity is given by

$$\sigma(\omega) \sim \omega\{\ln(\nu_{ph}/\omega)\}^\alpha \quad (32)$$

The parameter  $\nu_{ph}$  strongly depends on the localization parameter in the process involved and may even depend on temperature as well. Recently, this parameter has

been cleverly used to get scaled conductivity plot of chalcogenide glasses under various compositions[55]. More specifically, it is found that for  $(Li_2O)_{0.30} - (TeO_2)_{0.70}$  that a plot of  $\frac{\sigma_{real}}{\sigma_{dc}}$  as a function of  $\frac{\omega}{\nu_{ph}}$  falls into a single curve for all temperatures ranging between 383K and 483 K. Moreover, the plot of dc conductivity as a function of  $\frac{\omega}{\nu_{ph}}$  at fixed temperature with varying compositions yields a single scaled curve. These two graphs essentially have demonstrated the importance of  $\nu_{ph}$  as a scaling frequency.

In this Austin-Mott picture, we can easily write down the average jump time between the two localized states at R distance apart as

$$\frac{1}{\tau} = \nu_{ph} \exp(-2\alpha R) \exp(-\beta W(R)) \quad (33)$$

The most important contribution of hopping occurs at  $\omega\tau \sim 1$  which implies an emerging of a particular hopping length scale given by

$$R_\omega = \frac{1}{2\alpha} \ln(\nu_{ph}/\omega) \quad (34)$$

Within a given range of  $\Delta R$ , we also neglect the contribution of the hopping on the scale  $2\alpha d R_\omega \sim 1$ . A straightforward scaling analysis like non-interacting case[52] leads us to obtain

$$\sigma(\omega) \sim \omega T^{\nu+1} R_\omega^{-\frac{d\nu}{\nu+1}} R_\omega^{d+1} \quad (35)$$

The physical origin of the above terms is given below. The factor  $\omega$  comes from the disorder average of the Drude like term  $\frac{\omega^2\tau}{1+\omega^2\tau^2}$ . The  $T^{\nu+1}$  is from the typical average DOS calculation in a Fermi system at finite temperature. The first factor of the hopping term is from the energy difference of the localized states while the second one from the combination of phase space and the square of the dipole moment. The equation (35) can be further simplified to get the conductivity ( apart from the temperature variation) as

$$\sigma(\omega) \sim \omega \ln\left(\frac{\nu_{ph}}{\omega}\right)^{\alpha_1}, \quad \alpha_1(d, \nu) = \frac{d + \nu + 1}{\nu + 1} \quad (36)$$

This immediately indicates that

$$s(d, \nu, \omega) = 1 - \frac{\alpha_1}{\ln\left(\frac{\nu_{ph}}{\omega}\right)} \quad (37)$$

It is also evident from the expression of  $\sigma(\omega)$  that the equation (29) follows as a cross-check for non-interacting case in three dimensions ( $\nu = 0, d = 3$ ). In figure 3, we depict the double logarithmic plot of the A.C. conductivity as a function of frequency for both interacting as well as non-interacting case.

A comparison to the non-interacting case ( $\nu = 0$ ) suggests that

$$s_{int} > s_{non} \quad (38)$$

for *all spatial dimensions*  $d$ . The reason is the same as outlined in d.c. conductivity case or high electric field. It is also noted that  $\alpha_1$  satisfies the following inequality

$$\alpha_1(d, \nu) \geq 2 \quad (39)$$

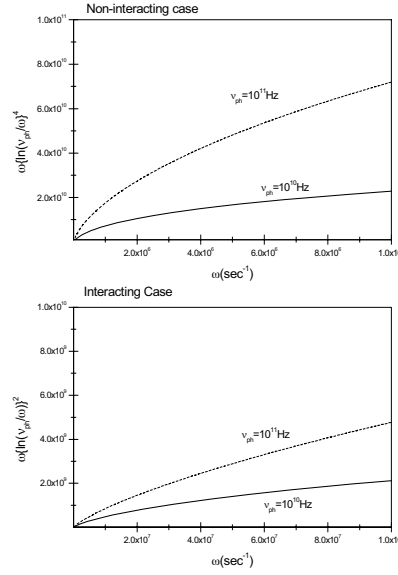


Figure 3: In this figure, the typical double logarithmic plot of 3d A.C. conductivity as a function of frequency in the interacting as well as non-interacting case is depicted for two  $\nu_{ph}$  frequencies.

It is interesting to note that this treatment does not give  $\sigma(\omega) = 0$  at  $\omega = 0$ . The typical variation of  $s$  as a function of  $\frac{\nu_{ph}}{\omega}$  in figure 4 shows that one can go below to the value of  $\frac{\nu_{ph}}{\omega}$  in compared to the non-interacting case. This is one of the interesting aspects of the a.c. conductivity of this system. It is also evident from the figure that lower value of  $\nu_{ph}$  ensures the lower value for  $s$  parameter. Since  $\nu_{ph} \gg \omega$ , we notice that for interacting as well as for non-interacting systems  $s(\omega) < 1$  within the range of scaled frequency  $\frac{\nu_{ph}}{\omega}$ . As this scaled frequency goes to infinity, the value  $s(\omega)$  slowly reaches the upper bound value of 1. In this sense, this behavior of  $s(\omega)$  is universal independent of the nature of the system. For a practical situation in amorphous systems, if one takes  $\nu_{ph} \sim 10^{12}$  Hz and  $\omega \sim 10^4$  Hz, it is found that for interacting ( $\nu = 2$ ) case  $s = 0.95$  while for non-interacting case  $s = 0.89$  in 3 dimension.

## 6 Experimental Verification of Interacting VRH Model

In this section, we take two examples from recent literatures to discuss the validity of interacting VRH model. The first example is the application of VRH in the Quantum Hall Effect (QHE) to explore the variation of the localization length with temperature. The Quantum Hall phenomena [56, 57] arises in a two dimensional hetero-structure at high magnetic field ( $B \sim 1 - 30T$ ) and at low temperature ( $\leq 4K$ ). It is basically a strongly interacting two dimensional disordered system in a strong magnetic field. A cautionary remark with this two dimensional electron gas (2DEG) system is that one has to be very careful to apply perturbation theory [58, 59] to understand this phenomenon. The transverse conductivity is quantized while there exists a dissipation-less conduction along the longitudinal direction. Mathematically, it can be stated as  $\sigma_{xx} \rightarrow 0$ ,  $\rho_{xx} \rightarrow 0$  but  $\sigma_{xy} = \tilde{\nu} \frac{e^2}{h}$ , where  $\tilde{\nu} = \frac{p}{q}$  is a

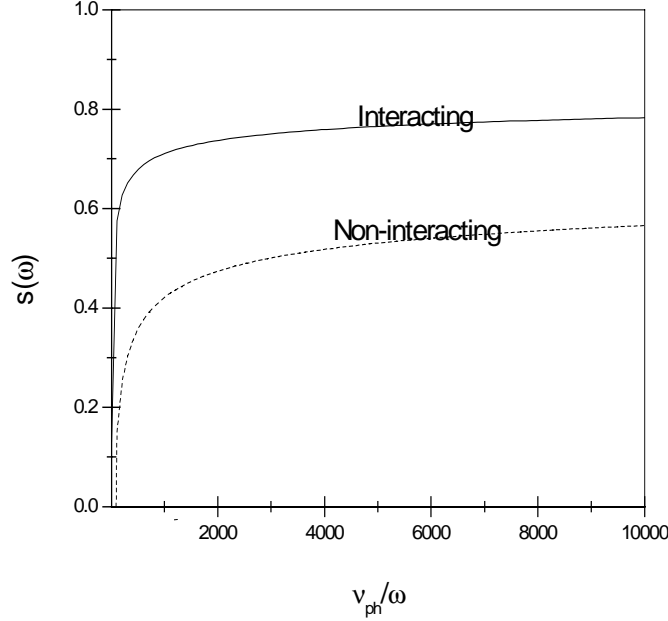


Figure 4: The typical slow variation of  $s$  as a function of frequency in 3d interacting case is shown in this figure. The typical variation of its counterpart is also indicated in the same plot.

rational fraction. This  $\tilde{\nu}$ , a dimensionless quantity is known as the filling factor and is given by  $\tilde{\nu} = 2\pi l^2 n = \frac{n\hbar c}{eB}$ , where  $n$  being the 2D density of electrons. With change of magnetic field, various plateaus are observed in the phase diagram of QHE. The transition between the adjacent plateaus forms an intriguing phenomenon [60, 61] due to the transport of the delocalized state within the Landau level center amidst localized states.

Polyakov and Shklovskii[62] have shown that in QHE regime, the VRH conductivity is given by

$$\sigma_{xx}(T) = \sigma_0 \exp\left(-\sqrt{\frac{T_0}{T}}\right), \quad k_B T_0 = C \frac{e^2}{4\pi\epsilon\epsilon_0\xi} \quad (40)$$

The pre-exponential factor  $\sigma_0$  depends inversely on temperature. The characteristic temperature  $T_0$ , however, as evident from the expression is controlled by the Coulomb energy at the localization length  $\xi$ .  $T_0$  essentially depends on  $\alpha$  (the inverse of localization length) and the density of states at the Fermi surface. Since  $\xi \propto \frac{1}{T_0}$ , it is possible to test the various scaling behavior near the edges of the plateau region. In other words, a plot of  $T\sigma_{xx}$  vs  $\frac{1}{\sqrt{T}}$  will scale the data collapse in a straight line. This was exactly what was verified recently[63] in doped GaAs/AlGaAs hetero-structures with electron density  $\sim 10^{15}m^{-2}$  and mobility  $\sim 10^2/Vs$ . The data were further tested for the typical Mott scaling in two dimensional case

$$\sigma_{xx} \propto T^{-m} \exp\left(-\left(\frac{T_0}{T}\right)^{1/3}\right) \quad (41)$$

with various values of  $m$ . But this Mott scaling ( $d = 2$  and  $\nu = 0$ ) did not show a good fitting to the experimental data. In other words, this immediately implies that in this situation Mott's *non-interacting* scaling is not suitable enough to describe the system. Besides this,

in the non-linear regime at high electric field, it was found through the measurements that the longitudinal conductivity does obey the scaling relation

$$\sigma_{xx} = \sigma_0^F(F) \exp\left(-\sqrt{\frac{F_0}{F}}\right), \quad F_0 \propto \frac{1}{\xi^2} \quad (42)$$

Note that in this regime, there is no explicit temperature dependence as per theoretical prediction. Thus, a good fitting of the data by the appropriate scaling laws clearly establishes the explicit *non-interacting* nature of the system under study.

As a second example, we consider the conduction measurement[20] of n-type CdSe nano crystal film as a function of temperature T and applied field F. It is known[32] that a crossover from Mott's  $\frac{1}{4}$  law to  $\frac{1}{2}$  occurs when Coulomb interaction is important at low enough temperature. This means that there exists[32] a critical temperature  $T_C$  given by

$$T_C = \frac{e^4 a g_0}{k_B (4\pi\epsilon\epsilon_0)^2} \quad (43)$$

above which one expects  $\frac{1}{4}$  law to obey. Here,  $a$  is taken as the typical localization length and  $g_0$  is the density of states at the Fermi surface. An estimation[64] of this  $T_C$  on CdSe QD thin films gives the value 400K which indicates  $\frac{1}{2}$  law instead of  $\frac{1}{4}$ . In fact, the low field measurement of this CdSe film in the temperature region of  $10K < T < 120K$  shows that the conductivity[20] satisfies the equation

$$G = A \exp\left(-\sqrt{\frac{T^*}{T}}\right) \quad (44)$$

The estimated  $T^*$  from the experimental measurement agrees reasonably well with the theoretical prediction[13]. Even the measurements carried out in high field ranging between  $10^5$  to  $10^7$  V/m at fixed temperature 4.3K indicate a temperature independent but strongly field dependent conductivity[20] given by

$$G = A \exp\left(-\sqrt{\frac{F^*}{F}}\right) \quad (45)$$

Thus, we notice that the experimental findings in the wide temperature region at low field as well as at high field region are well described by the Variable Range Hopping (VRH) model with a coulomb gap. Considering the Pseudo gap picture of interacting systems, it is clear that these two different systems correspond to  $d = 2$  and  $\nu = 1$ . In this way, it is possible to combine the various systems into a single conductivity class for each value of  $\nu$  according to classical phase transition idea. For example, Mott's well known  $\frac{1}{4}$  law falls into the category of  $d = 3$  and  $\nu = 0$ . Thus, with a given spatial dimension, one may easily identify a system according to a particular unique value of  $\nu$ . In this sense, this approach makes an important connection to statistical mechanics. Till now we have noticed that the system falls into either usual Mott category or Efros-Shklovskii one. It thus remains challenging as well as interesting to explore other universality classes of this generalized pseudo gap model in condensed matter physics.

## 7 Conclusions and Perspectives

To summarize, we have derived through a simple scaling analysis, a generalized form for the conductivity in an arbitrary dimension  $d$  with a density of states having a soft gap at the Fermi energy in amorphous systems. The behavior of the conductivity at high electric field has been obtained in this interacting system. A physical argument has been sought to understand the scaling behavior of the conductivity at high electric field. It is noticed that the exponents governing the behaviour of the conductivity in interacting cases are higher than that for non-interacting case. We have also given two examples from recent literature to show explicitly the validity of the interacting Variable Range Hopping (VRH). It will be of interest to look for the effect of pseudo gap in systems with an externally applied magnetic field (weak as well as strong). Work in this direction is in progress.

### Acknowledgements

The authors would like to thank the Editor of Nova Science Publishers for inviting to write this chapter. We have been benefitted from the stimulating discussions with Dr. Arvind Pan of IAMS, Taiwan. One (DJ) of us would like to acknowledge the computing facility of CCMS, National Taiwan University, Taiwan. We are also grateful to Prof. S. Kirkpatrick for helping us in finding some references in this area. This work is financially supported by UGC, New Delhi, India.

### References

- [1] P. W. Anderson, *Phys. Rev. B*, **109**, 1492 (1958).
- [2] N. F. Mott, *Phil. Mag. B*, **43**, 941 (1981).
- [3] N. F. Mott and E. A. Davies, *Electronic Processes in Non-Crystalline Materials*, (Oxford University Press, Oxford, U. K., second edition, 1979).
- [4] D. J. Thouless, in *Ill-Condensed Matter* edited by R. Balian, R. Maynard and G. Toulouse (North-Holland, Amsterdam, The Netherlands, 1979).
- [5] C. M. Soukoulis and E. N. Economou, *Phys. Rev. Lett.*, **52**, 565 (1982).
- [6] M. Schrieber, *Phys. Rev. B*, **31**, 6146, (1985).
- [7] B. Krammer, Y. Ono and T. Ohtsuki, *Surf. Sci.*, **196**, 127 (1988).
- [8] Y. Ono, T. Ohtsuki and B. Krammer, *J. Phys. Soc. Jpn.*, **58**, 1705 (1989).
- [9] D. Jana, *Indian J. Phys.*, **74A**, 547 (2000).
- [10] R. Peierls, *Z. Phys.*, **80**, 763 (1933).
- [11] S. R. Elliot, *Physics of Amorphous Materials*, (Longman, London, U.K., 1984).

- 
- [12] R. Zallen, *The Physics of Amorphous Solids*, (Wiley-Interscience, New York, USA, 1998).
- [13] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors*, (Springer-Verlag, Berlin, 1984).
- [14] N. F. Mott, *Phil. Mag.*, **19**, 835 (1969).
- [15] P. A. Lee and T. V. Ramakrishna, *Rev. Mod. Phys.*, **57**, 287 (1985).
- [16] D. Belitz and T. R. Kirkpatrick, *Rev. Mod. Phys.*, **66**, 261 (1994).
- [17] T. G. Castner, *Phys. Rev. B*, **61**, 16596 (2000).
- [18] A. Y. Rogatchev and U. Misutani, *Phys. Rev. B*, **61**, 15550 (2000).
- [19] J. K. Schön, C. Kloc and B. Batlogg, *Phys. Rev. B*, **63**, 5304 (2001).
- [20] Dong Yu, Congjun Wang, Brian L. Wehrenberg, Philippe Guyot-Sionnet, *Phys. Rev. Lett.*, **92**, 216802 (2004).
- [21] S. Ebisu, T. Sogabe, M. Hayashi and S. Nagata, *J. Phys. Chem. Solids.*, **61**, 869 (2000).
- [22] V. A. Samuilov, J. Galibert, V. K. Ksenevich, V. J. Goldman, M. Rafailovich, J. Sokolov, I. A. Bashmarov and V. A. Dorosinets, *Physica B*, **294**, 319 (2001).
- [23] Z. H. Wang, C. Li, E. M. Scherr, A. G. McComick and A. J. Epstein, *Phys. Rev. Lett.*, **66**, 1475 (1991).
- [24] A. A. Reedijk, H. C. F. Martens, H. B. Brom, M. A. J. Michels, *Phys. Rev. Lett.*, **83**, 3904 (1999).
- [25] T. Vuletic, B. Korun-Harnzic, S. To,ic, B. Gorshunov, P. Haas, M. Dressel, J. Akir-nitsu, T. Sasaki, T. Nagata, *Phys. Rev. B*, **67**, 184521 (2003).
- [26] P. Lunkenheimer, G. Knebel, A. Pimenov, G. A. Emel'chenko, A. Loidl, *Z. Phys. B*, **99**, 507 (1996).
- [27] V. Ambegaokar, B. I. Halperin and J. S. Langer, *Phys. Rev.* **B4**, 2612 (1971).
- [28] S. Kirkpatrick, *Rev. Mod. Phys.*, **45**, 574 (1973).
- [29] A. Miller and S. Abrahams, *Phys. Rev.*, **120**, 745 (1960).
- [30] M. J. Hirsch, *Comments Cond. Mat. Phys.*, **13**, No. 5, 249 (1987).
- [31] M. L. Knotek and M. Pollak, *Phil. Mag.*, **35**, 1133 (1976).
- [32] A. L. Efros and B. I. Shklovskii, *J. Phys. C*, **8**, L49 (1975).
- [33] J. H. Davies, P. A. Lee and T. M. Rice, *Phys. Rev. B*, **29**, 4240 (1984).
- [34] N. F. Mott, *Contemp. Phys.*, **26**, 203 (1985).



- [35] S. Lamba and D. Kumar, *Phys. Rev. B*, **59**, 4752 (1999). V. Malik and D. Kumar, *Phys. Rev. B*, **69**, 153103 (2004).
- [36] M. Pollak and M. Ortuno, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak ( North-Holland, Amsterdam, 1985), p. 287.
- [37] A. L. Efros and B. I. Shklovskii, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak ( North-Holland, Amsterdam, 1985), p. 409.
- [38] M. Ghosh, A. K. Meikap, S. K. Chattopadhyay and S. Chatterjee, *J. Phys. Chem. Solids*, **62**, 475 (2001).
- [39] D. Jana and J. Fort, *Physica B*, **344**, 62 (2004).
- [40] S. Teber, *Variable-range hopping in electronic crystals*, <http://arXiv.org/abs/cond-mat/0404449>.
- [41] A. L. Efros, *J. Phys. C*, **9**, 2021 (1976).
- [42] A. J. Zabrodskii, *Sov. Phys. Semiconduct.*, **11**, 345 (1977).
- [43] M. M. Fogler, S. Teber and B. I. Shklovskii, *Phys. Rev. B*, **69**, 035413 (2004).
- [44] T. F. Rosenbaum, K. Andres and G. A. Thomas, *Solid State Commun.*, **35**, 663 (1980).
- [45] J. Joo, S. M. Long, J. P. Pouget, E. J. Oh, A. G. MacDiarmid and A. J. Epstein, *Phys. Rev. B*, **57**, 9567 (1998).
- [46] N. Apsley and H. P. Huges, *Phil. Mag.*, **30**, 963 (1974); *Phil. Mag.*, **31**, 1327 (1975).
- [47] R. M. Hill, *Phil. Mag.*, **24**, 1307 (1971).
- [48] S. Kirkpatrick, in *Amorphous and Liquid Semiconductors* edited by J. Stuke and W. Brenig, ( Taylor and Francis, 1974), p. 183.
- [49] G. Amow, N. P. Raju and J. E. Greedan, *J. Solid State Chem.*, **155**, 177 (2000).
- [50] Throughout this paper we have assumed that the characteristic phonon frequency  $\nu_{ph}$  is independent of  $R$  and  $W$ . However, according to Miller and Abraham[29] model,  $\nu_{ph}$  depends on  $R$  and  $W$ . In that situation, an additional temperature dependence of  $T^{\frac{d-2(\nu+1)}{d+\nu+1}}$  should be included in the conductivity expression. We note that in such a situation  $B_{MA}(d, \nu) \leq B(d, \nu)$ . This indicates a weak temperature dependence of the pre-exponential factor in Miller-Abraham case than Mott case.
- [51] Using the bounds on  $\nu$ , it is observed that the exponent  $B_{MA}$  of the pre-exponential factor in Miller-Abraham case however satisfies the inequality  $d - \frac{5}{2} \leq B_{MA} < \nu$ .
- [52] I. G. Austin and N. F. Mott, *Adv. Phys.*, **18**, 41 (1969).
- [53] M. Pollak, *Phys. Rev. A*, **133**, 564 (1964).

- 
- [54] A. Ghosh and A. Pan, *Phys. Rev. Lett.*, **84**, 2188 (2000).
- [55] A. K. Jonscher, *Nature* (London), **256**, 566 (1975).
- [56] R. E. Prange and S. M. Girvin, *The Quantum Hall Effect* ( Springer, Berlin, 1987).
- [57] T. Chakraborty and P. Pietiläinen, *The Fractional Quantum Hall Effect*, Springer-Verlag, Berlin, New York, 1988. For a modern view points see S. M. Girvin, *The Quantum Hall Effect: Novel Excitations and Broken Symmetries*, <http://arXiv.org/abs/cond-mat/9907002>.
- [58] D. Jana, *Physics Teacher*, **44**, 21 (2002).
- [59] S. C. Zhang, *Int. J. Mod. Phys. B*, **6**, 25 (1992).
- [60] B. Huckestein, *Rev. Mod. Phys.*, **67**, 357 (1995).
- [61] S. L. Sondhi, S. M. Girvin, J. P. Carini and D. Shahar, *Rev. Mod. Phys.*, **69**, 315 (1997).
- [62] D. G. Polyakov and B. I. Shklovskii, *Phys. Rev. B*, **48**, 11167 (1993); *Phys. Rev. Lett.*, **70**, 3796 (1993).
- [63] F. Hohls, U. Zeitler and R. J. Hang, *Phys. Rev. Lett.*, **88**, 036802 (2002).
- [64] P. Guyot-Sionnest and C. Wang, *J. Phys. Chem. B*, **107**, 7355 (2003).