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Measurable temperatures in nonequilibrium radiative systems

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Abstract

Information theory yields different predictions for the radiation intensity of nonequilibrium radiative systems, depending on what quantity one identifies with the temperature. A possible experiment is proposed that may, in principle, be used in order to determine whether the generalized temperature of extended irreversible thermodynamics is the measurable temperature in nonequilibrium radiative systems or not.

1. Introduction

In thermodynamic equilibrium, the entropy S of any system is a function of extensive variables, e.g. of the internal energy U and the volume V. The temperature T of the system can be introduced phenomenologically through $T^{-1} = \partial S/\partial U$ [1]. Under the local-equilibrium hypothesis, the local entropy per unit mass s depends on the corresponding quantities per unit mass, e.g. on the specific internal energy u and the specific volume v, and the temperature can be introduced phenomenologically as $T^{-1} = \partial s/\partial u$ [2]. In the same way as local-equilibrium thermodynamics generalizes the analysis of equilibrium systems in order to cope with near-equilibrium states, the possibility to generalize local-equilibrium thermodynamics has been considered many times. The experimental and conceptual limitations of the local-equilibrium approach, for example for fast phenomena or systems characterized by long relaxation times, are well known (see, e.g., Ref. [3] and references therein). Some of the limitations in question may

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be avoided in the framework of extended irreversible thermodynamics (EIT) [3–7]. Within this approach, the local-equilibrium hypothesis does not hold in general and the specific entropy depends not only on the local-equilibrium independent variables (such as u and v) but also on the dissipative fluxes, such as the conductive heat flux $\boldsymbol{q}$. By analogy with the thermodynamics of equilibrium and local-equilibrium systems, it seems reasonable to introduce the temperature phenomenologically through $T^{-1} = \partial s/\partial u$. However, this is an open problem, because such a procedure has deep consequences. For example, for a classical nonrelativistic monatomic ideal gas it is found [3,8] that outside local-equilibrium T does not coincide with the kinetic "temperature" T_m, which satisfies that

$$u = \frac{3k_B T_m}{2m}\,, \tag{1}$$

with k_B the Boltzmann constant and m the molecular mass.

This topic may at first sight seem a purely academic exercise (or, even, simply a notational discussion). However, it has been predicted phenomenologically that, in general, the heat flux is proportional to ∇T, rather than to ∇T_m, and this is a strong point for the claim that T, rather than T_m, is the measurable temperature [9,3]. More generally, since the properties of macroscopic systems depend on their temperature and EIT yields different results for T and T_m, for a given value of the temperature, obtained by the experimental reading of a thermometer, the predicted properties will be different if we assume that this value corresponds to T than if we assume it corresponds to T_m. In fact, several experiments have been recently proposed in order to check whether T is the quantity measured by a thermometer in nonequilibrium or not [9–12]. In the case of a highly photoexcited plasma in semiconductors, it has even been found that comparison of theory and experiment seems to point to the conclusion that the temperature T of EIT is the quantity measured by a thermometer [12]. However, more research on this topic is certainly necessary, both theoretically and experimentally, before a definite conclusion can be reached. It would, of course, be desirable to address the problem in a variety of nonequilibrium systems. In the present paper, we will tackle the problem making use of a recent information-theoretical approach to nonequilibrium radiative systems [13], which was motivated in part by the fact that, in contrast with matter [14], the radiation distribution function is very easy to observe directly by means of an spectrophotometer. We shall see that, within such a statistical approach, the predictions for the radiation intensity show observable differences depending on whether one assumes that the quantity measured by a thermometer is the temperature T of EIT or the local-equilibrium temperature of the considered system. An important difference between the analysis presented here and those referred above is that we will consider the question of the generalized temperature for radiative systems: the heat flux is due to radiation instead of matter. We will see that in this case new features arise, but not only with regards to the temperature of EIT: even the expression for the local-equilibrium temperature must be reformulated. In this way, the problem of the measurable temperature outside equilibrium will be shown to be more general and its importance will become even more clear than previously expected. The plan of the paper is as follows: in Section 2, we sum up some previous results that are necessary for the present discussion. In Section 3, we compare the predictions for the radiation intensity in the case that the temperature of EIT is assumed to be the measurable temperature with those in the case that the local-equilibrium temperature is assumed to be measurable for arbitrarily far-from-equilibrium states. Section 4 is devoted to some concluding remarks.

2. Summary of information-theoretical results for the radiation intensity

We consider a system composed of matter and radiation under a radiative heat flux $\boldsymbol{F}$. One such simple system is a cavity with highly absorbing internal walls, one or several small apertures (see Fig. 1) and containing a classical nonrelativistic ideal gas composed of monatomic molecules. As in Ref. [13], heat conduction and convection are ignored (i.e., we assume the matter heat flux $\boldsymbol{q}$ and barycentric velocity to be negligible). If the temperature of the system were uniform, there would be no net radiative heat flux ($\boldsymbol{F} = 0$) and the intensity of radiation emitted through the apertures would be Planckian. We consider the more general case in which the temperature may depend on the vertical coordinate z. In this case it is clear from considerations of symmetry that $\boldsymbol{F} = (0, 0, F)$, with $F \neq 0$, in general. In order to prevent convective instabilities, we assume that the temperature decreases downwards (so that in Fig. 1 the temperature at the aperture B is higher than that at the aperture A). We choose the positive vertical direction downwards, so that $F > 0$. We propose to generalize the usual EIT concept of nonequilibrium temperature to this system, which is composed of both matter and radiation, by defining

$$T^{-1} = \frac{\partial(s_m + s_r)}{\partial(u_m + u_r)}, \tag{2}$$

with s_m and u_m the specific matter entropy and internal energy, respectively, and s_r and u_r the radiation entropy and energy, also per unit mass of matter. Maximization of the total (i.e., radiation and matter) entropy density of the system under the constraints of given total energy density, matter number density and radiative heat flux yields for the radiation intensity, after use of the steady-state gray radiative transfer equation and assuming that the gradient of T is uniform [13],

$$\begin{aligned} I_\nu = I_\nu^{(2)} + O(\varepsilon^3) = I_{\nu\,\mathrm{Planck}}(T)[1 + \tilde{\phi}^{(1)}(T, \nabla T)\cos\theta \\ + \tilde{\phi}^{(2)}(T, \nabla T)\cos^2\theta] + O(\varepsilon^3), \end{aligned} \tag{3}$$

where θ is the angle between the direction corresponding to I_ν and the positive z-direction (the downwards vertical direction in Fig. 1), and $I_{\nu\mathrm{Planck}}$ the Planck function, namely,

$$I_{\nu\mathrm{Planck}}(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_B T} - 1}, \tag{4}$$

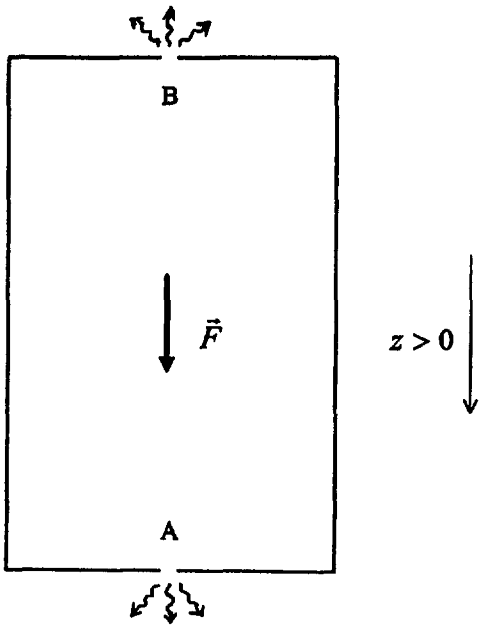

Fig. 1. Experiment proposed in order to determine which quantity corresponds to the measurable temperature in radiative nonequilibrium states. The figure represents a cavity with highly absorbing internal walls and containing an ideal gas. Radiation may leave the cavity through the small apertures A and B. The temperature is assumed to decrease in the vertical downwards direction. $\boldsymbol{F}$ is the radiative heat flux.

with h the Planck constant, ν the radiation frequency and c the velocity of light in vacuo. The first- and second-order corrections to the Planckian intensity are, respectively,

$$\tilde{\phi}^{(1)}(T,\nabla T)=\frac{h\nu}{k_B\kappa T^2}\frac{e^{h\nu/k_BT}}{e^{h\nu/k_BT}-1}\left|\frac{dT}{dz}\right|, \tag{5}$$

$$\tilde{\phi}^{(2)}(T,\nabla T)=\frac{1}{2}\frac{h^2\nu^2}{k_B^2\kappa^2T^4}\frac{e^{h\nu/k_BT}+1}{(e^{h\nu/k_BT}-1)^2}e^{h\nu/k_BT}\left(\frac{dT}{dz}\right)^2, \tag{6}$$

with κ the absorption coefficient. Eqs. (3)–(6) were obtained in Ref. [13] by performing a MacLaurin expansion up to second order in a parameter ε. This parameter satisfies that $0\leqslant\varepsilon\leqslant 1$ ($\varepsilon = 0$ corresponding to equilibrium and $\varepsilon = 1$ to the free-streaming case), it is a measure of how far away from equilibrium the system is, and may be written as (see Ref. [13] or Appendix A to the present paper)

$$\boldsymbol{\varepsilon}=(0,0,\varepsilon)=-\frac{1}{\kappa T}\nabla T\,. \tag{7}$$

Photons interact with matter with a mean-free path $l = 1/\kappa$ [15], so that $\varepsilon = (l/T)|dT/dz|$. It turns out that this expression is exactly the same as that obtained for the smallness parameter which is used in the Chapman–Enskog kinetic theory of heat conduction in gases. Since in this case there is experimental evidence from ultrasound propagation and shock waves that ε is, indeed, a valid nonequilibrium expansion parameter [16], we consider that: (i) it seems reasonable to make use of this smallness parameter in heat radiation, and (ii) the second-order approximation (3) should break down for high enough values of ε. We also mention that the first-order approximation, i.e., $I_\nu \approx I_{\nu\text{Planck}}(T)[1+\tilde{\phi}^{(1)}(T,\nabla T)\cos\theta]$ with $\tilde{\phi}^{(1)}(T,\nabla T)$ given by Eq. (5), coincides with the results of near-equilibrium diffusion theory, which is an approximate description of nonequilibrium radiative systems that had been previously derived on phenomenological grounds [15,17].

In order to compare Eqs. (3)–(6) to experimental measurements, one must first of all take into account that a spectrophotometer does not measure the intensity of radiation for a given direction, but the intensity of all photons that cross a unit area coming from all possible directions of a hemisphere. For this reason, and considering, for example, the apertures A and B in Fig. 1, the directly measurable intensities are

$$i_{\nu A} = \int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta \sin\theta \cos\theta \, I_{\nu A} \,, \tag{8}$$

$$i_{\nu B} = -\int_0^{2\pi} d\varphi \int_{\pi/2}^{\pi} d\theta \sin\theta \cos\theta \, I_{\nu B} \,. \tag{9}$$

Finally, use is made of Eqs. (3)–(6) to integrate Eqs. (8) and (9) and the results are expressed per unit wavelength ($\lambda = c/\nu$), instead of per unit frequency. This yields [13]

$$i_{\lambda\mathrm{A}} = i_{\lambda\mathrm{A}}^{(2)} + O(\varepsilon_\mathrm{A}^3) = i_{\lambda\mathrm{Planck}}(T_\mathrm{A})[1+\phi_\lambda^{(1)}(T_\mathrm{A},\nabla T)+\phi_\lambda^{(2)}(T_\mathrm{A},\nabla T)] + O(\varepsilon_\mathrm{A}^3) \,, \tag{10}$$

$$i_{\lambda\mathrm{B}} = i_{\lambda\mathrm{B}}^{(2)} + O(\varepsilon_\mathrm{B}^3) = i_{\lambda\mathrm{Planck}}(T_\mathrm{B})[1-\phi_\lambda^{(1)}(T_\mathrm{B},\nabla T)+\phi_\lambda^{(2)}(T_\mathrm{B},\nabla T)] + O(\varepsilon_\mathrm{B}^3) \,, \tag{11}$$

with

$$i_{\lambda\mathrm{Planck}}(T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/k_B T\lambda}-1} \,, \tag{12}$$

$$\phi_\lambda^{(1)}(T,\nabla T) = \frac{2}{3}\frac{hc}{k_B \kappa T^2 \lambda}\frac{e^{hc/k_B T\lambda}}{e^{hc/k_B T\lambda}-1}\left|\frac{dT}{dz}\right| , \tag{13}$$

$$\phi_\lambda^{(2)}(T,\nabla T) = \frac{1}{4}\frac{h^2c^2}{k_B^2\kappa^2T^4\lambda^2}\frac{e^{hc/k_B T\lambda}+1}{(e^{hc/k_B T\lambda}-1)^2}e^{hc/k_B T\lambda}\left(\frac{dT}{dz}\right)^2 . \tag{14}$$

In order to compare these expressions with experimental data, it is necessary to address another item, namely whether the EIT temperature T is the quantity measured by a thermometer or not. Note that T has been introduced by means of Eq. (2). However, there are several possible ways to introduce a temperature-like variable for the system. One may in principle consider T_m, defined by Eq. (1), or one may also follow the usual approach in radiative transfer, namely to introduce a quantity T_r through [17]

$$u_r = aT_r^4 . \tag{15}$$

For the system under consideration we have [13]

$$T_m = T , \tag{16}$$

$$T_r = T(1 + \tfrac{5}{6}\varepsilon^2) + O(\varepsilon^3) . \tag{17}$$

Eq. (16) would not hold in the presence of heat conduction or convection [3,8]. On the other hand, it has been argued [17,13] that T_r is just a parameter related to the radiation part of the system, but, in general, has no thermodynamical meaning. In equilibrium we have $\varepsilon = 0$ and therefore $T = T_m = T_r$, so that Eq. (2) implies that $T_r^{-1} = \partial(s_m + s_r)/\partial(u_m + u_r)$. These results are valid in equilibrium. They would also hold in case all equilibrium relations were assumed to hold locally, but their validity breaks down in general nonequilibrium states (see Eq. (17)). Therefore, T_r may be called the local-equilibrium temperature of the considered system. The explicit expression for the entropy and the generalized Gibbs equation can be found out, and this yields a formalism that closely parallels that of the EIT theory of conductive and convective systems [18]. However, here we would like to go beyond the conceptual discussions considered so far and will show that the problem of whether T or T_r is the quantity measured by a thermometer can, in principle, be solved experimentally.

3. A proposal in order to determine experimentally which quantity is measured by a thermometer

The second-order intensity, Eqs. (3)–(6), is written in terms of T. If we want to compare the measurable predictions of the theory with those that would result from the assumption that the local-equilibrium radiative temperature T_r (instead of T) is the measurable temperature, we must write the intensity in terms of T_r instead of T. In order to do so, we need to express $\boldsymbol{\varepsilon}$ in terms of T_r and its gradient, just as Eq. (7) relates $\boldsymbol{\varepsilon}$ with T and its gradient. Eq. (7) was derived in Ref. [13]. A different derivation is included in Appendix A to the present paper, where it is shown that $\boldsymbol{\varepsilon}$ may also be written, in the same order of approximation, as

$$\boldsymbol{\varepsilon} = (0, 0, \varepsilon) = -\frac{1}{\kappa T_r}\nabla T_r . \tag{18}$$

We make use of this result and of Eq. (17) into Eqs. (3)–(6). After neglecting higher-order terms we obtain that the expansion, Eqs. (3)–(6) may be written in terms of T_r as

$$I_\nu = I_{\nu r}^{(2)} + O(\varepsilon^3) = I_{\nu\mathrm{Planck}}(T_r)[1 + \tilde{\phi}_r^{(1)}(T_r, \nabla T_r)\cos\theta + \tilde{\phi}_r^{(2)}(T_r, \nabla T_r)\cos^2\theta + \tilde{\tilde{\phi}}_r^{(2)}(T_r, \nabla T_r)] + O(\varepsilon^3)\,, \tag{19}$$

with

$$I_{\nu\mathrm{Planck}}(T_r) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT_r}-1}\,, \tag{20}$$

$$\tilde{\phi}_r^{(1)}(T_r, \nabla T_r) = \frac{h\nu}{k_B\kappa T_r^2}\frac{e^{h\nu/k_BT_r}}{e^{h\nu/k_BT_r}-1}\left|\frac{dT_r}{dz}\right|, \tag{21}$$

$$\tilde{\phi}_r^{(2)}(T_r, \nabla T_r) = \frac{1}{2}\frac{h^2\nu^2}{k_B^2\kappa^2T_r^4}\frac{e^{h\nu/k_BT_r}+1}{(e^{h\nu/k_BT_r}-1)^2}e^{h\nu/k_BT_r}\left(\frac{dT_r}{dz}\right)^2. \tag{22}$$

$$\tilde{\tilde{\phi}}_r^{(2)}(T_r, \nabla T_r) = -\frac{5}{6}\frac{h\nu}{k_B\kappa^2T_r^3}\frac{e^{h\nu/k_BT_r}}{e^{h\nu/k_BT_r}-1}\left(\frac{dT_r}{dz}\right)^2, \tag{23}$$

We note that the presence of $\tilde{\tilde{\phi}}_r^{(2)}(T_r, \nabla T_r)$ in Eq. (19) implies that $I_{\nu r}^{(2)}$ is a function of T_r and ∇T_r that differs from the function $I_\nu^{(2)}$ of T and ∇T (see Eqs. (3)–(6)). This has the consequence that, if at a given point in Fig. 1 we measure, e.g., a temperature of 2000 K and an absolute temperature gradient of 15 K/m, we will obtain different predictions for the intensity if we assume that T is measurable temperature (i.e., $T =$ 2000 K and $|dT/dz| = 15$ K/m) and apply $I_\nu^{(2)}$, obtained from Eqs. (3)–(6), than if we assume that T_r is measurable temperature (i.e., $T_r = 2000$ K and $|dT_r/dz| = 15$ K/m) and apply $I_{\nu r}^{(2)}$, obtained from Eqs. (19)–(23). Let us assume, e.g., that $\kappa = 10\,\mathrm{m}^{-1}$ (see Ref. [13]). Then in the first case ($T = 2000$ K) we would have, according to Eq. (7), that $\varepsilon = 0.075$ and Eq. (17) yields $T_r \approx 2009$ K, whereas in the second case ($T_r = 2000$ K) we obtain from Eqs. (18) and (17) that $T \approx 1991$ K. Thus, the differences between T and T_r are less than 0.5%. However, we shall see that even such small differences lead to differences between the predictions for the directly measurable intensities that are higher than 2%. This could provide an experimental way to determine what quantity is the measurable temperature. We also mention that the result, Eqs. (19)–(23), for the radiation intensity in terms of T_r is analogous to the information-theoretical nonrelativistic matter distribution function in terms of T_m that has been derived previously (see Eqs. (30)–(34) in Ref. [19]).

Making use of Eqs. (19)–(23) into Eqs. (8) and (9) we obtain, instead of Eqs. (10)–(14),

$$i_{\lambda\mathrm{A}} = i_{\lambda r\mathrm{A}}^{(2)} + O(\varepsilon_\mathrm{A}^3) = i_{\lambda\mathrm{Planck}}(T_{r\mathrm{A}})[1 + \phi_{\lambda r}^{(1)}(T_{r\mathrm{A}}, \nabla T_r) + \phi_{\lambda r}^{(2)}(T_{r\mathrm{A}}, \nabla T_r)] + O(\varepsilon_\mathrm{A}^3)\,, \tag{24}$$

$$i_{\lambda\mathrm{B}} = i_{\lambda r\mathrm{B}}^{(2)} + O(\varepsilon_\mathrm{B}^3) = i_{\lambda\mathrm{Planck}}(T_{r\mathrm{B}})[1 - \phi_{r\lambda}^{(1)}(T_{r\mathrm{B}}, \nabla T_r) + \phi_{r\lambda}^{(2)}(T_{r\mathrm{B}}, \nabla T_r)] + O(\varepsilon_\mathrm{B}^3)\,, \tag{25}$$

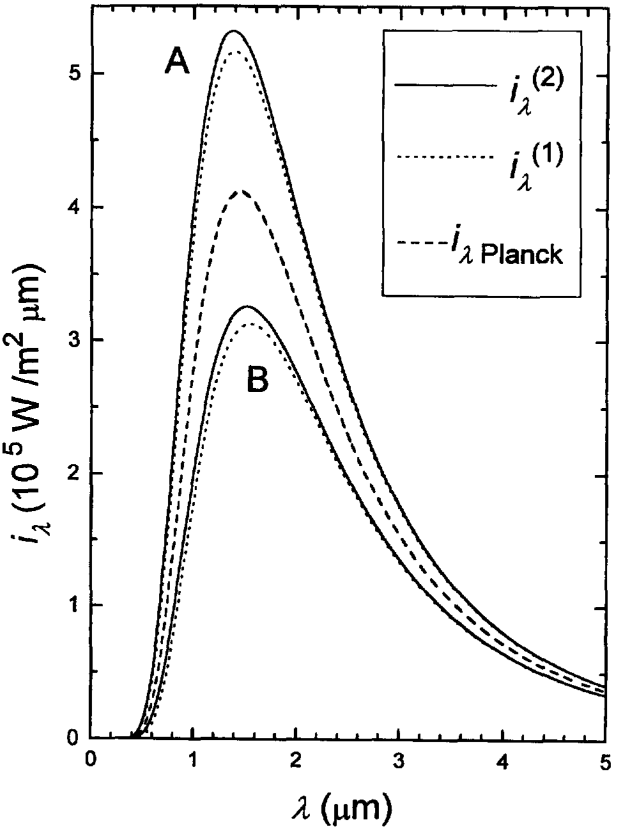

Fig. 2. The two full lines in this figure correspond to the spectra of radiation emitted by the cavity shown in Fig. 1 through the apertures A and B, according to the second-order approximation and assuming that the generalized temperature T of EIT is the measurable temperature. It has been assumed that $T_A = 2000\,K$, $T_B = 2001\,K$, $|dT/dz| = 15\,K/m$ and $\kappa = 10\,m^{-1}$. Planckian and first-order spectra (dashed and dotted lines, respectively) are included for comparison.

with

$$i_{\lambda\text{Planck}}(T_r) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/k_B T_r \lambda} - 1}\,, \tag{26}$$

$$\phi_{r\lambda}^{(1)}(T_r, \nabla T_r) = \frac{2}{3} \frac{hc}{k_B \kappa T_r^2 \lambda} \frac{e^{hc/k_B T_r \lambda}}{e^{hc/k_B T_r \lambda} - 1} \left| \frac{dT_r}{dz} \right|, \tag{27}$$

$$\phi_{r\lambda}^{(2)}(T_r, \nabla T_r) = \frac{1}{2} \frac{hc}{k_B \kappa^2 T_r^3 \lambda} \frac{1}{e^{hc/k_B T_r \lambda} - 1} \times \left[-\frac{5}{3} e^{hc/k_B T_r \lambda} + \frac{hc}{2 k_B T_r \lambda} \frac{e^{hc/k_B T_r \lambda} + 1}{e^{hc/k_B T_r \lambda} - 1} e^{hc/k_B T_r \lambda} \right] \left(\frac{dT_r}{dz} \right)^2 . \tag{28}$$

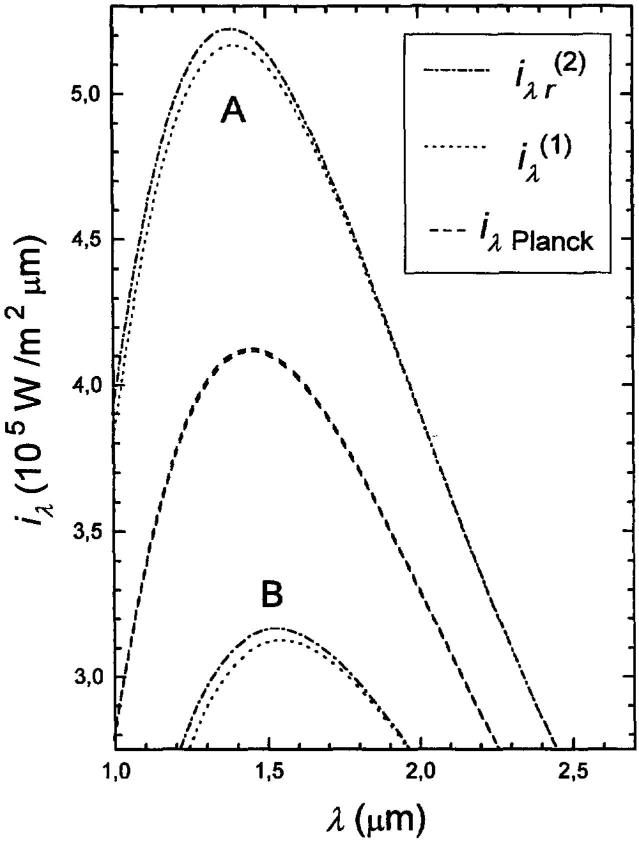

Fig. 3. The same as in Fig. 2, but assuming that the local-equilibirum temperature T_r, instead of T, is the quantity measured by a thermometer. It has been assumed that $T_{rA} = 2000\,\mathrm{K}$, $T_{rB} = 2001\,\mathrm{K}$, $|dT_r/dz| = 15\,\mathrm{K/m}$ and $\kappa = 10\,\mathrm{m}^{-1}$. As in Fig. 2, the Planckian or zeroth-order spectra corresponding to both apertures in Fig. 1 are barely distinguishable from each other (dashed lines).

In order to carry out some estimations, let us assume that the measured temperatures in Fig. 1 are of 2000 K in point A and 2001 K in point B, that the absolute temperature gradient is 15 K/m (so that the distance between A and B would be of 6.7 cm) and, as in Ref. [13], that $\kappa = 10\,\mathrm{m}^{-1}$. Making use of these values, we will obtain different predictions depending on whether we assume that T or T_r is the measurable temperature:

(i) Assuming that T is the measurable temperature, we have $T_A = 2000\,\mathrm{K}$, $T_B = 2001\,\mathrm{K}$ and $|dT/dz| = 15\,\mathrm{K/m}$. The predicted spectra for the radiation emitted through apertures A and B, obtained from Eqs. (10)–(14), are plotted in Fig. 2. In addition to $i_\lambda^{(2)}$, equilibrium and first-order spectra are also included in Fig. 2 for comparison. The zeroth order or equilibrium spectra, denoted by $i_{\lambda\mathrm{Planck}}$ in Fig. 2, are not distinguishable from each other at this scale (the dashed line in Fig. 2 consists, in fact, of two

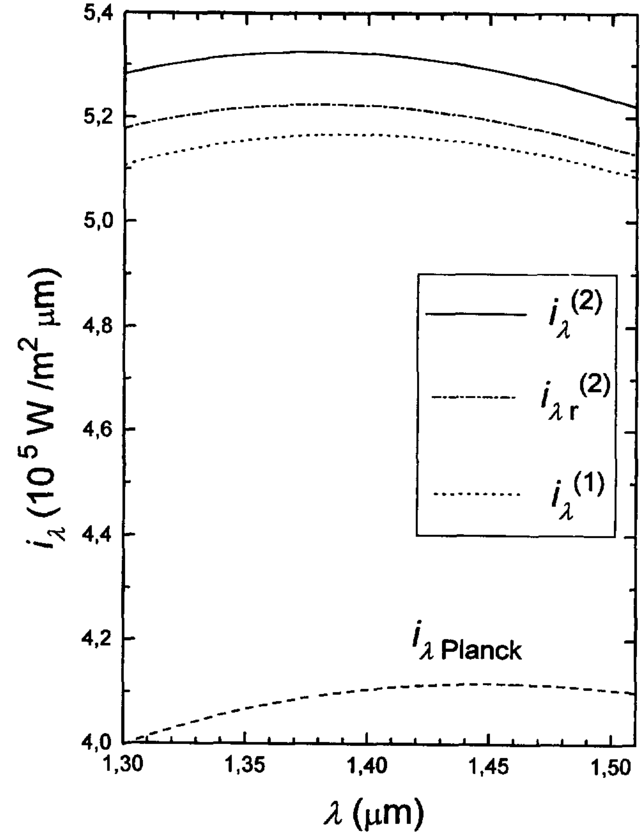

Fig. 4. Those spectra from Figs. 2 and 3 that correspond to the aperture A are reproduced here in a narrower wavelength range. This allows to estimate the differences between the second-order spectra, predicted by information theory in case T is assumed to be the measurable temperature (full-line) with those predicted in case T_r is assumed to be the measurable temperature (dashed–dotted line).

closely similar spectra). The first-order spectra correspond to $i_\lambda^{(1)} = i_{\lambda\mathrm{Planck}}(T)[1 + \phi_\lambda^{(1)}(T, \nabla T)]$.

(ii) Assuming that T_r is the measurable temperature, we have $T_{r\mathrm{A}} = 2000\,\mathrm{K}$, $T_{r\mathrm{B}} = 2001\,\mathrm{K}$ and $|dT_r/dz| = 15\,\mathrm{K/m}$. The predicted spectra for the radiation emitted through apertures A and B, obtained from Eqs. (24)–(28), are plotted in Fig. 3. Because in this case the second-order corrections are smaller than in the previous one, a different scale has been used in the figure. Of course, the zeroth- and first-order spectra are the same in both figures, because Eq. (12) is the same function of T as Eq. (26) is of T_r, and Eq. (13) is the same function of T and $|dT/dz|$ as Eq. (27) is of T_r and $|dT_r/dz|$. The differences in the predictions arise in the second-order theory, and are due to the fact that Eq. (14) is not the same function of T and $|dT/dz|$ as Eq. (28) is of T_r and $|dT_r/dz|$. In order to evaluate these differences, in Fig. 4 we plot the same spectra for the aperture A as in Figs. 2 and 3 (the main conclusions would not change if the aperture B were considered). The spectra in Fig. 4 have been plotted around a narrow wavelength interval near the maxima (we may note that Wien's displacement law does not hold outside equilibrium, but this interesting point is discussed elsewhere [20]). From Fig. 4 we find that the first-order correction reaches a value of about 26%. On the other hand, the second-order intensity yields a correction to the Planckian intensity of up to about 30% if T is assumed to be the measurable temperature, but only of about 27% if T_r is assumed to be the quantity measured by a thermometer. More precisely, the observable differences between both assumptions with respect to the Planckian spectra are of about 2.5%. The relative difference, i.e. $(i_\lambda^{(2)} - i_{\lambda r}^{(2)})/i_{\lambda r}^{(2)}$, reaches a value of about 2%. Such a difference is not negligible and could therefore, in principle, make it possible to determine which quantity, T or T_r, is the measurable temperature.

4. Concluding remarks

We have proposed an experiment, depicted in Fig. 1, that may be useful in order to determine experimentally whether the generalized temperature of EIT, T, is the measurable temperature outside local-equilibrium or not. The information-theoretical predictions for the nonequilibrium deviations of the intensity with respect to the Planckian spectrum depend on whether one assumes that the EIT temperature, T, or the local-equilibrium temperature, T_r, is the quantity measured by a thermometer. This leads to differences higher than 2% with respect to the equilibrium, or Planckian, results.

Of course, still higher differences could have been obtained by assuming higher values of the temperature gradient. However, in this case the second-order corrections would be considerably increased in comparison with the first-order ones (the first- and second-order corrections are linear and quadratic in the temperature gradient, respectively, see Eqs. (13)–(14) and Eqs. (27)–(28)), and third-order terms (which are difficult to calculate explicitly) would become increasingly important.

In closing this paper, and with the intention to give a complete discussion of the mathematical formalism, we outline that in writing Eqs. (19)–(23) from Eqs. (3)–(6), third- and higher-order terms have been neglected. Thus, it is important to check that the neglected terms do not substantially affect the difference between the estimations obtained from both sets of expressions. We will illustrate how we have done this by means of a specific example. For $T_A = 2000\,\mathrm{K}, |dT/dz| = 15\,\mathrm{K/m}$ and $\kappa = 10\,\mathrm{m}^{-1}$ we have, according to Eqs. (17) and (7), that $T_{Ar} \approx 2009\,\mathrm{K}$, and find from these values and Eqs. (10)–(14), (24)–(28) and (A.14) that the difference between $i_\nu^{(2)}$ and $i_{\nu r}^{(2)}$ is only of about 0.36%. This shows that the neglected third-order terms do not substantially change the estimations performed, so that the conclusion that important differences arise from the assumption that T, instead of T_r, is the measurable temperature, remains unchanged.

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Appendix A. Derivation of Eqs. (7) and (18)

In this appendix we will first present a derivation of Eq. (7) in the second-order approximation. In contrast with the method used in Ref. [13], the one presented below will be very useful for the purposes of the present paper, because a similar procedure will allow us to show the validity of Eq. (18) in the same order of approximation.

It has been shown that an information-theoretical approach leads to the following second-order expressions (see Eqs. (B.3) and (B.1) in Ref. [13])

$$\boldsymbol{F} = \frac{4acT^4}{3}\boldsymbol{\varepsilon} + O(\varepsilon^3)\,, \tag{A.1}$$

$$P_{rzz} = \frac{aT^4}{3}(1 + 6\varepsilon^2) + O(\varepsilon^3)\,, \tag{A.2}$$

with $\boldsymbol{F}$ the radiative heat flux, $\boldsymbol{\varepsilon} = (0, 0, \varepsilon)$ and $\boldsymbol{P}_r$ the radiation pressure tensor. For the sake of simplicity [13], $\boldsymbol{F}$ has been assumed to be parallel to the z axis. In Appendix B in Ref. [13], third-order terms in the expansion for $\boldsymbol{F}$ were taken into account. We shall now see that it is, in fact, possible to derive Eq. (7) without doing so.

It is well known that in the case under consideration, the gray steady-state radiative transfer equation leads to the relation (see Refs. [21,22], or Eq. (25) in Ref. [13])

$$c\frac{\partial P_{rzz}}{\partial z} = -\kappa F\,. \tag{A.3}$$

As in Ref. [13], we consider a special, the simplest possible case, by assuming situations such that: (i) both T and $\boldsymbol{\varepsilon}$ depend only on the z coordinate; (ii) $\nabla T = (0, 0, dT/dz)$ is uniform; and (iii) the absorption coefficient κ is uniform. Substitution of Eqs. (A.1) and (A.2) into Eq. (A.3) yields

$$\frac{1}{3T}\frac{dT}{dz} + \frac{2\varepsilon^2}{T}\frac{dT}{dz} + \varepsilon\frac{d\varepsilon}{dz} = -\frac{\kappa}{3}\varepsilon\,. \tag{A.4}$$

In order to find out an expression for ε, we may follow two different approaches that yield the same result. We can make use of the method of successive approximations [13] and neglect terms of third and higher order (we can do so because we are working in the second-order approximation). Or we may simply check, by direct substitution into Eq. (A.4), that

$$\boldsymbol{\varepsilon} = -\frac{1}{\kappa T}\ \nabla T \tag{A.5}$$

satisfies Eq. (A.4). This is so because Eq. (A.5) implies that the second and third terms in the left-hand side of Eq. (A.4) are negligible in the second-order theory: we have $\varepsilon^2 dT/dz = -\kappa T\varepsilon^3$, which is of third order in ε; and we also have, making use of assumptions (ii) and (iii), that

$$\varepsilon\frac{d\varepsilon}{dz} = \kappa\varepsilon^3\,, \tag{A.6}$$

again a third-order result.

Eq. (A.5) completes the derivation of Eq. (7) in the present paper. As mentioned above, this result had been obtained previously (Eq. (B.8) in Ref. [13]). However, the derivation presented here shows that there is no need to make use of the method of successive approximations, because some terms that seem relevant at first sight are seen to be of higher order by use of Eq. (A.5) as an *ansatz*. This method will now be shown to provide a simple derivation of Eq. (18) in the present paper.

Eqs. (A.1) and (A.2) are second-order expressions for $\boldsymbol{F}$ and P_{rzz} written in terms of T and $\boldsymbol{\varepsilon}$. In terms of T_r, instead of T, we have (see Eqs. (35) and (36) in Ref. [13])

$$\boldsymbol{F} = \frac{4acT_r^4}{3}\boldsymbol{\varepsilon} + O(\varepsilon^3)\,, \tag{A.7}$$

$$P_{rzz} = \frac{aT_r^4}{3}\left(1+\frac{8}{3}\varepsilon^2\right) + O(\varepsilon^3)\,. \tag{A.8}$$

Making use of these two equations into Eq. (A.3) we obtain, instead of Eq. (A.4),

$$\frac{1}{T_r}\frac{dT_r}{dz} + \frac{8\varepsilon^2}{3T_r}\frac{dT_r}{dz} + \frac{4}{3}\varepsilon\frac{d\varepsilon}{dz} = -\kappa\varepsilon\,, \tag{A.9}$$

where the last term in the left-hand side is negligible in the second-order approximation (see Eq. (A.6)). Therefore,

$$\varepsilon = -\frac{1}{\kappa T_r}\left(1+\frac{8}{3}\varepsilon^2\right)\frac{dT_r}{dz}\,. \tag{A.10}$$

At this point we may follow several procedures. One of them is analogous to the last step in the derivation of Eq. (A.5): it is simple to check that the *ansatz*

$$\boldsymbol{\varepsilon} = -\frac{1}{\kappa T_r}\nabla T_r \tag{A.11}$$

satisfies Eq. (A.10): the second term in the right-hand side of Eq. (A.10) is then easily checked to be of third order in ε. Eq. (A.11) is the result to be derived (i.e., Eq. (18)).

There is a different derivation that makes the previous *ansatz* unnecessary and seems interesting. In order to present it, we shall first prove that, in the present level of approximation, we have $dT/dz = dT_r/dz$. From Eqs. (17) and (A.5) we find, within the second-order theory,

$$T_r = T + \frac{5}{6\kappa^2 T}(dT/dz)^2 \,. \tag{A.12}$$

From this equation and the assumptions, already applied above, that dT/dz and κ are uniform, we obtain

$$\frac{dT_r}{dz} = \frac{dT}{dz} - \frac{5}{6\kappa^2 T^2}\left(\frac{dT}{dz}\right)^3 . \tag{A.13}$$

The last term in Eq. (A.13) is of third order in ε (see Eq. (A.5)). Therefore, in the second-order theory we have

$$\frac{dT}{dz} = \frac{dT_r}{dz} \,. \tag{A.14}$$

Making use of Eqs. (17) and (A.14) into Eq. (A.5) we obtain

$$\varepsilon = -\frac{1}{\kappa T_r}\left(1 + \frac{5}{6}\varepsilon^2\right)\frac{dT_r}{dz} \,. \tag{A.15}$$

Combining this equation with Eq. (A.10) we obtain that $\varepsilon^2 dT_r/dz$ is negligible in the second-order theory, i.e., that it is at least of third order. By taking this into account, Eq. (A.15) immediately yields Eq. (18) or Eq. (A.11), as it should.

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