



Information-theoretical derivation of a nonequilibrium extension of Wien's displacement law*

J. Fort^{a,1}, J.A. González^a, J.E. Llebot^b

^a *Secció de Física, Departament d'Enginyeria Industrial, Escola Politècnica Superior, Universitat de Girona, Avda. Lluís Santaló s/n, 17071 Girona, Catalonia, Spain*

^b *Grup de Física, Departament de Ciències Ambientals, Facultat de Ciències, Universitat de Girona, Plaça de l'Hospital 6, 17071 Girona, Catalonia, Spain*

Received 28 August 1997; accepted for publication 25 September 1997

Communicated by J. Flouquet

Abstract

Wien's displacement law for the wavelength of maximum intensity of a blackbody is extended to nonequilibrium systems. A non-vanishing temperature gradient causes a correction in the displacement law that is predicted and estimated making use of information theory. A simple application is presented. © 1997 Elsevier Science B.V.

PACS: 05.30.-d; 52.25.Rv; 44.40.+a; 87.50.Hj

1. Introduction

Wien's displacement law refers to blackbodies in equilibrium. It states that the maximum intensity emitted by such blackbodies occurs at a wavelength that is proportional to the inverse of the temperature. The proportionality constant can be written in terms of three fundamental constants (the speed of light in vacuo c , the Planck constant h and the Boltzmann constant k) and follows directly from Planck's radiation law. Planck's law is independent of the nature of the blackbody: it depends only on its temperature. Of course, the same happens for Wien's displacement law. This conceptually appealing subject also has very

interesting applications: Wien's displacement law and Planck's spectral law are useful in systems where the radiation intensity is high enough to be detectable but the temperature is difficult or impossible to measure by means of a contact thermometer. A classical application is the determination of the surface temperatures of stars [1]. Another application is sonoluminescence, a very active topic of research, in which it has been proposed that comparison of the measured spectra with equilibrium (or Planckian) ones can be used in order to evaluate the temperature, and therefore make it possible to test different theoretical models [2]. Wien's displacement law and Planck's spectral law also have very important applications in radiation thermometry [3,4]. However, since Planck's law applies only to equilibrium states, which imply a uniform temperature distribution in the system, it immediately raises the question of its generalization to nonequilibrium states. This is an interesting topic in itself, and

* Dedicated to Professor Jose Casas-Vazquez on the occasion of his 60th anniversary and of the 25th anniversary of the publication of the first paper on EIT by his UAB research group.

¹ E-mail: jfort@songoku.udg.es.

also from the point of view of applications. In fact, it is well known that both the outer layers of a star [5] and the bubbles generated by ultrasonic waves [6] are far from equilibrium. Departure from equilibrium is also observed in many systems, such as calibration sources and industrial furnaces, that are of importance in connection to radiation thermometry [3].

A well-known generalization of Planck's radiation law to nonequilibrium states is the so-called near-equilibrium diffusion theory. This theory applies to states near thermodynamic equilibrium and was originally derived on phenomenological grounds [7,8]. Recently, it has also been derived from statistical mechanics, making use of information theory [9]. Such an approach provides a method for the theoretical description of arbitrarily far-from-equilibrium radiative systems. In the first nonequilibrium approximation, this method yields the results of near-equilibrium diffusion theory, the second approximation has also been worked out explicitly and it yields a generalization of near-equilibrium diffusion to states further away from equilibrium. Our purposes are here to derive nonequilibrium extensions of Wien's displacement law from the first-order (or near-equilibrium diffusion) and second-order theories, and also to analyze under which conditions the near-equilibrium diffusion displacement law may be trusted, making use of the second-order information theory.

2. Information-theoretical extension of Wien's displacement law

We consider the simple radiative system depicted in Fig. 1. It consists of an enclosure that contains a gas and has highly absorbing internal walls and one or several small apertures. We consider situations in which the temperature depends on the vertical coordinate z . Then the system is not in equilibrium and the radiation intensity *per unit solid angle* is no longer Planckian, not even isotropic (this is analogous to the fact that the matter distribution function of an ideal gas under a temperature gradient is neither Maxwellian nor isotropic [10]). In order to prevent convective effects, we assume that the temperature decreases downwards in the system depicted in Fig. 1. In Ref. [9], it has been proposed that such a system may be used in order to test experimentally the predictions of ra-

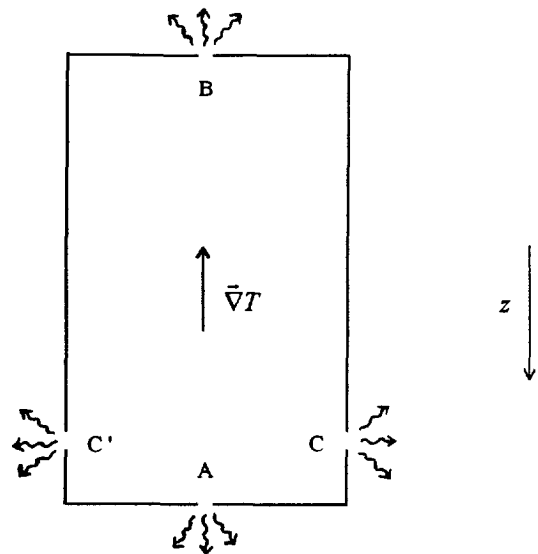


Fig. 1. Cavity with highly absorbing internal walls and containing a gas. Radiation may leave the cavity through apertures A, B, C, and C'. In the present Letter the displacement law is studied for steady states such that there is a temperature gradient in the direction shown in the figure

diative transfer theoretical models, and information-theoretical nonequilibrium spectra for the radiation leaving the cavity through the apertures in Fig. 1 were calculated and displayed for some specific cases (for the sake of simplicity, it was assumed that the material inside the enclosure is a classical monatomic ideal gas). In this way it was observed, for a very particular case, that Wien's displacement law is predicted to break down outside equilibrium (see Fig. 3b in Ref. [9] and comments therein). In view of the applications that we have recalled here (see the first paragraph in the previous section), it is convenient to try to approach this problem in general, not only because of its conceptual interest but also because of practical motivations. In order to do so, we will first briefly reproduce some necessary previous results, which were derived under the assumptions that the system is in a steady state and that the temperature gradient is uniform. Such assumptions make the problem easier to handle mathematically. We mention that the underlying statistical-mechanical approach can also be generalized to non-steady states, leading to the inclusion of radiative transfer in a wider thermodynamical framework [11]. Here we are interested in the displace-

ment law for the radiation intensity. For this purpose it is important to note that, whereas in equilibrium the same results for the displacement law are obtained independently of whether one considers the maximum of the radiation intensity i_λ or of the energy density u_λ (both of them are here defined per unit wavelength), different displacement laws will be obtained for both quantities outside equilibrium. This is due to the fact that the well-known equilibrium relationship, namely $i_\lambda = (c/4)u_\lambda$, is valid only for isotropic radiation. Therefore, outside equilibrium we must study the radiation intensity i_λ (which can be directly measured by means of a spectrophotometer) and not the radiation energy density u_λ (which cannot be measured experimentally). The information-theoretically predicted intensities, due to all photons that leave the enclosure in Fig. 1 through the aperture A, B, C or C', can be found after integration of the radiation intensities per unit solid angle over the appropriate ranges of solid angle, and are respectively (see Eqs. (62)–(66) and (38) in Ref. [9])

$$i_{\lambda A} = i_{\lambda \text{Planck}}(T_A) [1 + \phi_{\lambda 1}(T_A, \varepsilon_A) + \phi_{\lambda 2}(T_A, \varepsilon_A)] + O(\varepsilon_A^3), \tag{1}$$

$$i_{\lambda B} = i_{\lambda \text{Planck}}(T_B) [1 - \phi_{\lambda 1}(T_B, \varepsilon_B) + \phi_{\lambda 2}(T_B, \varepsilon_B)] + O(\varepsilon_B^3), \tag{2}$$

$$i_{\lambda C} = i_{\lambda C'} = i_{\lambda \text{Planck}}(T_C) [1 + \frac{1}{2}\phi_{\lambda 2}(T_C, \varepsilon_C)] + O(\varepsilon_C^3), \tag{3}$$

with

$$i_{\lambda \text{Planck}}(T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{\exp(hc/kT\lambda) - 1}, \tag{4}$$

$$\phi_{\lambda 1}(T, \varepsilon) = \frac{2}{3} \frac{ch}{kT\lambda} \frac{\exp(hc/kT\lambda)}{\exp(hc/kT\lambda) - 1} \varepsilon, \tag{5}$$

$$\phi_{\lambda 2}(T, \varepsilon) = \frac{1}{4} \frac{c^2 h^2}{k^2 T^2 \lambda^2} \times \frac{[\exp(hc/kT\lambda) + 1] \exp(hc/kT\lambda)}{[\exp(hc/kT\lambda) - 1]^2} \varepsilon^2, \tag{6}$$

where c , h , and k have already been defined in Section 1, T is the temperature, λ is the wavelength and ε is a small parameter (its range of allowed values is $0 \leq \varepsilon \leq 1$ [9]) that may be written, in the case and order of approximation considered, as

$$\varepsilon = \frac{1}{\sigma T} \left| \frac{dT}{dz} \right|, \tag{7}$$

with σ the absorption coefficient of the gas contained in the enclosure. For the sake of simplicity, this coefficient is assumed to be wavelength dependent (this is called the gray approximation and is very convenient in radiative transfer in order to get relatively simple expressions [7,12]). Because the mean free path of a photon is $l = 1/\sigma$ [7], Eq. (7) may be written as $\varepsilon = (l/T)|dT/dz|$. This is precisely the small parameter used in the Enskog kinetic theory method for conductive (instead of radiative) systems (the so-called Knudsen number), and in this case there is wide experimental support for its validity as a nonequilibrium expansion parameter [13]. For given values of l and T , the higher the temperature gradient is, the higher the value of ε will be, and this will make it necessary to consider more terms in the expansions (1)–(3), so that the radiation will show larger deviations from the Planckian spectra, corresponding to the fact that the system is further away from equilibrium.

In thermodynamic equilibrium, the temperature is uniform and we have (see Eq. (7)) $\varepsilon = 0$ at all points of the system, so that all of the spectra (1)–(3) are Planckian with the same temperature, as they should.

In the first-order approximation, second- and higher-order terms in the small parameter ε are neglected; thus, the intensities under consideration are given, instead of Eqs. (1)–(3), by

$$i_{\lambda A}^{(1)} = i_{\lambda \text{Planck}}(T_A) [1 + \phi_{\lambda 1}(T_A, \varepsilon_A)], \tag{8}$$

$$i_{\lambda B}^{(1)} = i_{\lambda \text{Planck}}(T_B) [1 - \phi_{\lambda 1}(T_B, \varepsilon_B)], \tag{9}$$

$$i_{\lambda C}^{(1)} = i_{\lambda C'}^{(1)} = i_{\lambda \text{Planck}}(T_C), \tag{10}$$

with $i_{\lambda \text{Planck}}(T)$ and $\phi_{\lambda 1}(T, \varepsilon)$ given by Eqs. (4) and (5), respectively. As shown in Ref. [9], these first-order results also follow from near-equilibrium diffusion theory [7,8].

Since we are interested, in the first place, in finding the wavelengths such that the near-equilibrium diffusion intensity of the radiation leaving the cavity through each aperture in Fig. 1 reaches its maximum, we must set the derivatives of Eq. (8)–(10) with respect to the wavelength equal to zero. This yields, respectively,

$$5 + (x_A - 5) \exp(x_A) = 2\varepsilon_A \frac{x_A \exp(x_A)}{\exp(x_A) - 1} \times [-2 - \frac{1}{3}x_A + (2 - \frac{1}{3}x_A) \exp(x_A)], \quad (11)$$

$$5 + (x_B - 5) \exp(x_B) = -2\varepsilon_B \frac{x_B \exp(x_B)}{\exp(x_B) - 1} \times [-2 - \frac{1}{3}x_B + (2 - \frac{1}{3}x_B) \exp(x_B)], \quad (12)$$

$$5 + (x_C - 5) \exp(x_C) = 0, \quad (13)$$

where we have introduced the dimensionless quantities

$$x_A \equiv \frac{ch}{kT_A \lambda_{\max A}}, \quad x_B \equiv \frac{ch}{kT_B \lambda_{\max B}},$$

$$x_C \equiv \frac{ch}{kT_C \lambda_{\max C}}, \quad (14)$$

with $\lambda_{\max A}$, $\lambda_{\max B}$ and $\lambda_{\max C}$ the wavelength of maximum intensity for the radiation emitted by the enclosure through the aperture A, B, and C (or C'), respectively.

In thermodynamic equilibrium we have, as recalled above, $\varepsilon_A = 0$, $\varepsilon_B = 0$ and $\varepsilon_C = 0$, and in this special case Eqs. (11)–(13) reduce, as they should, to the well-known [14] equilibrium equation $(1 - \frac{1}{5}x)e^x = 1$, with $x \equiv x_A = x_B = x_C$ (we stress that this is valid in the absence of thermal inhomogeneities). The solution to this equation can be found making use of numerical methods and is $x = 4.9651$ [14,15].

We observe from Eq. (13) that, within the first-order approximation, the wavelength of maximum intensity for the apertures C and C' is the same as in equilibrium. This is due to the fact that the first-order intensity (10) (which we stress has been integrated over the appropriate range of solid angle) is Planckian. In contrast, the first-order correction to the intensity corresponding to apertures A and B does not vanish (see Eqs. (8) and (9)). This causes a first-order correction to the Wien displacement law for the radiation leaving the enclosure through apertures A and B: the right-hand sides of Eqs. (11) and (12) do not vanish identically, in contrast to that of Eq. (13).

We have recalled that, even in equilibrium, numerical methods are necessary in order to find explicit expressions for the wavelength of maximum intensity. Similarly, we have only succeeded in dealing with the solutions to the first-order nonequilibrium equations

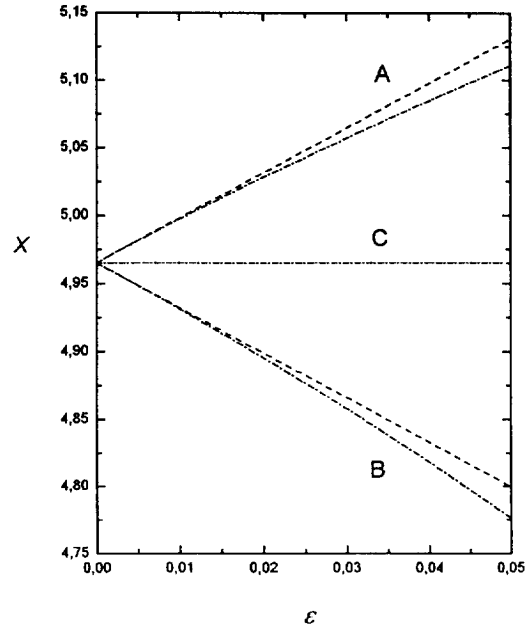


Fig. 2. First-order predictions for the displacement law. The vertical axis x stands for x_A , x_B and x_C for the aperture A, B, and C (or C') in Fig. 1 respectively. These quantities are related to the wavelength of maximum intensity by Eq. (14). The horizontal axis corresponds to ε_A for x_A , ε_B for x_B and ε_C for x_C . The dashed-dotted lines are numerical solutions to Eqs. (11)–(13). The dashed lines correspond to linear approximations given by Eqs. (17)–(19). For the aperture C (and C'), both lines are the same.

(11)–(13) numerically. In Fig. 2 we present the numerical solutions to these equations (dashed-dotted lines). From this figure we observe the following. As states further away from equilibrium are considered, there is an increase in the value of x_A (which corresponds to a decrease of $\lambda_{\max A}$, see Eq. (14)) and a decrease in the value of x_B (which corresponds to an increase of the value of $\lambda_{\max B}$). Thus, we conclude that in near-equilibrium states, the first-order (or near-equilibrium diffusion) theory predicts a decrease of the wavelength of maximum intensity for an observer who measures the intensity looking along the direction of the temperature gradient (i.e., an observer located at the aperture A in Fig. 1) and an increase of the wavelength of maximum intensity for an observer who looks in the direction opposite to that of the temperature gradient (i.e., an observer located at B). However, in such near-equilibrium states no detectable modification of the wavelength of maximum

intensity is predicted for an observer who measures the radiation looking in a direction orthogonal to that of temperature gradient (apertures C and C' in Fig. 1). As we shall see, much modification is predicted only for states further away from equilibrium. On the other hand, it would not be difficult to find predictions corresponding to other directions, but the ones considered here are enough in order to illustrate the predicted effect.

Because Eqs. (11) and (12) are rather complicated, it is interesting to find approximate expressions. This can be achieved in the following way. Consider, e.g., aperture A. We look for the first-order MacLaurin expansion corresponding to the function $x_A(\varepsilon_A)$, i.e. we look for the linear approximation to the dash-dotted line A in Fig. 2. We denote this approximation by $x_A^{(1)}(\varepsilon_A)$. Thus

$$x_A^{(1)}(\varepsilon_A) = x_A|_{\varepsilon_A=0} + \left. \frac{dx_A}{d\varepsilon_A} \right|_{\varepsilon_A=0} \varepsilon_A, \tag{15}$$

with $x_A|_{\varepsilon_A=0} = 4.9651$. A simple way to find the value of $dx_A/d\varepsilon_A|_{\varepsilon_A=0}$ is the following. We first write Eq. (11) as

$$\begin{aligned} & [\exp(x_A) - 1][5 + (x_A - 5)\exp(x_A)] \\ &= 2\varepsilon_A x_A \exp(x_A) \\ & \times \left[-2 - \frac{1}{3}x_A + \left(2 - \frac{1}{3}x_A\right)\exp(x_A)\right]. \end{aligned}$$

It is easy to apply the usual method of implicit derivation [16] to this equation. In this way, we can find an expression for $dx_A/d\varepsilon_A$ and evaluate this derivative at $\varepsilon_A = 0$. This yields, after applying that $5 + (x_A - 5)\exp(x_A) = 0$ for $\varepsilon_A = 0$ (see Eq. (11)),

$$\begin{aligned} \left. \frac{dx_A}{d\varepsilon_A} \right|_{\varepsilon_A=0} &= [2x_A \exp(x_A) \\ & \times \left[-2 - \frac{1}{3}x_A + \left(2 - \frac{1}{3}x_A\right)\exp(x_A)\right] \\ & \times \{[\exp(x_A) - 1][\exp(x_A) - 5]\}^{-1}|_{x_A=4.9651} \\ &= 3.3101, \end{aligned} \tag{16}$$

so that we have, making use of Eq. (15),

$$x_A^{(1)}(\varepsilon_A) = 4.9651 + 3.3101\varepsilon_A. \tag{17}$$

For aperture B, the same procedure yields

$$x_B^{(1)}(\varepsilon_B) = 4.9651 - 3.3101\varepsilon_B, \tag{18}$$

whereas for apertures C and C' we have

$$x_C^{(1)}(\varepsilon_C) = x_{C'}^{(1)}(\varepsilon_{C'}) = 4.9651. \tag{19}$$

In order to avoid confusion, let us mention explicitly that we have $\varepsilon_A = \varepsilon_{C'}$. This follows directly from Eq. (7), the fact that apertures C and C' have the same value of the z-coordinate (see Fig. 1), and the assumptions that the temperature depends only on the z-coordinate and that the temperature gradient is uniform.

Fig. 2 also includes the approximate solutions (17)–(19) (dashed lines). They are seen to be very good approximations for values of the nonequilibrium parameter ε smaller than 0.02: then the differences with respect to the numerical solutions to Eqs. (11) and (12) (dashed-dotted lines) are less than 0.1%. It might at first sight seem that second-order MacLaurin expansions (instead of the first-order ones of Eqs. (17)–(19)) would yield better approximations. However, it should be noted that second-order terms have already been neglected in Eqs. (8)–(10), so it is clear that such a procedure would be senseless from a physical point of view. In order to find better approximations than Eqs. (17)–(19), we have to make use of the second-order intensities (1)–(3). By neglecting terms of third and higher order, and requiring the vanishing of the derivatives of these expressions with respect to the wavelength we obtain, instead of Eqs. (11)–(13),

$$\begin{aligned} 5 + (x_A - 5)\exp(x_A) &= 2\varepsilon_A \frac{x_A \exp(x_A)}{\exp(x_A) - 1} \\ & \times \left[-2 - \frac{1}{3}x_A + \left(2 - \frac{1}{3}x_A\right)\exp(x_A)\right] \\ & + \frac{1}{4}\varepsilon_A^2 \frac{x_A^2 \exp(x_A)}{[\exp(x_A) - 1]^2} \\ & \times [-7 - x_A - 4x_A \exp(x_A) + (7 - x_A)\exp(2x_A)], \end{aligned} \tag{20}$$

$$\begin{aligned} 5 + (x_B - 5)\exp(x_B) &= -2\varepsilon_B \frac{x_B \exp(x_B)}{\exp(x_B) - 1} \\ & \times \left[-2 - \frac{1}{3}x_B + \left(2 - \frac{1}{3}x_B\right)\exp(x_B)\right] \\ & + \frac{1}{4}\varepsilon_B^2 \frac{x_B^2 \exp(x_B)}{[\exp(x_B) - 1]^2} \\ & \times [-7 - x_B - 4x_B \exp(x_B) + (7 - x_B)\exp(2x_B)], \end{aligned} \tag{21}$$

$$5 + (x_C - 5) \exp(x_C) = \frac{1}{8} \varepsilon_C^2 \frac{x_C^2 \exp(x_C)}{[\exp(x_C) - 1]^2} \times [-7 - x_C - 4x_C \exp(x_C) + (7 - x_C) \exp(2x_C)], \quad (22)$$

In Fig. 3 we plot the numerical solutions to these three equations (full lines). We find that the relative differences between the numerical solutions to the first and second-order implicit equations (dashed-dotted and full lines, respectively) are less than 0.1% for values of ε smaller than 0.02. Therefore, the near-equilibrium diffusion generalizations of the Wien displacement equation (Eqs. (11)–(13) and their approximate solutions (17)–(19)) should be reasonably accurate in a given point of the system, provided that the temperature, temperature gradient and absorption coefficient are such that the value of ε , given by Eq. (7), is smaller than 0.02.

Simpler results than Eqs. (20)–(22) can be found by determining the corresponding second-order MacLaurin series. For example, for the aperture A we define the function

$$x_A^{(2)}(\varepsilon_A) = x_A|_{\varepsilon_A=0} + \left. \frac{dx_A}{d\varepsilon_A} \right|_{\varepsilon_A=0} \varepsilon_A + \frac{1}{2} \left. \frac{d^2x_A}{d\varepsilon_A^2} \right|_{\varepsilon_A=0} \varepsilon_A^2. \quad (23)$$

An implicit derivation of Eq. (20) leads to Eq. (16), as it should. By implicitly deriving twice, the second derivative can be found. The analytical result is rather lengthy, so that we will only reproduce the value we are interested in, namely its numerical evaluation at the origin,

$$\left. \frac{d^2x_A}{d\varepsilon_A^2} \right|_{\varepsilon_A=0} = 7.1416. \quad (24)$$

We therefore see that the first-order expansion (17) is generalized by

$$x_A^{(2)}(\varepsilon_A) = 4.9651 + 3.3101\varepsilon_A + 3.5708\varepsilon_A^2. \quad (25)$$

Following the same procedure for apertures B and C (or C') we find, respectively,

$$x_B^{(2)}(\varepsilon_B) = 4.9651 - 3.3101\varepsilon_B + 3.5708\varepsilon_B^2, \quad (26)$$

$$x_C^{(2)}(\varepsilon_C) = x_{C'}^{(2)}(\varepsilon_{C'}) = 4.9651 + 6.1383\varepsilon_C^2. \quad (27)$$

In Figs. 3a,b we have included the second-order series (25) and (26) (dotted lines). For values of ε up to

0.05, it is seen that these yield reliable approximations, as compared with the first-order ones (17) and (18), to the numerical solutions of Eqs. (20) and (21) (full lines). Similarly, in Fig. 3c we have included a plot of the second-order approximate solution (27) (dotted line), but in this case we have chosen a range of ε_C up to 0.25, because for the aperture C it is seen that this solution is rather close to the numerical solution to Eq. (22) (full line) even for $\varepsilon_C = 0.25$: The relative difference is only about 0.4%, whereas the near-equilibrium diffusion approximation (dashed-dotted line) corresponds to a difference, again relative to the second-order exact result (full-line), of about 6.8%. Of course, the second-order series (25)–(27) can be extrapolated to higher values of ε , but third and higher-order terms (which would be very difficult to evaluate) would become increasingly important.

3. Simple application

In order to check the usefulness of the results presented, we would like to consider the displacement law for a specific case. Let us assume that a small part of some physical system (which could be an outer layer of a star [1] or of a sonoluminescent bubble [2], or the material in an industrial furnace [3], etc.) can be approximately represented by Fig. 1 (see, e.g., Ref. [17]) and that we observe the radiation of the system looking in the direction of the temperature gradient (i.e., we consider the radiation crossing the aperture A in Fig. 1). If the values of the relevant parameters are, e.g., $T_A = 2000$ K, $\sigma = 0.1 \text{ m}^{-1}$ (see Ref. [9]) and $|dT/dz| = 10 \text{ K/m}$, what are the wavelengths of maximum intensity predicted by the different approximations? The simplest approach would be to apply the usual equilibrium laws to this nonequilibrium system, i.e. to assume that the intensity is approximately Planckian. Under this zeroth-order approximation (see Eq. (1)), one would simply make use of the Wien displacement law. As explained under Eq. (14), this yields $x_A = 4.9651$ and therefore, making use of Eq. (14), one would obtain $\lambda_{\max A} = ch/kT_A x_A = 1.4489 \mu\text{m}$. In contrast, we may take into account that the system under consideration is a nonequilibrium system and begin with the calculation of the nonequilibrium small parameter: Eq. (7) yields $\varepsilon_A = 0.05$. Then the numerical solution to the

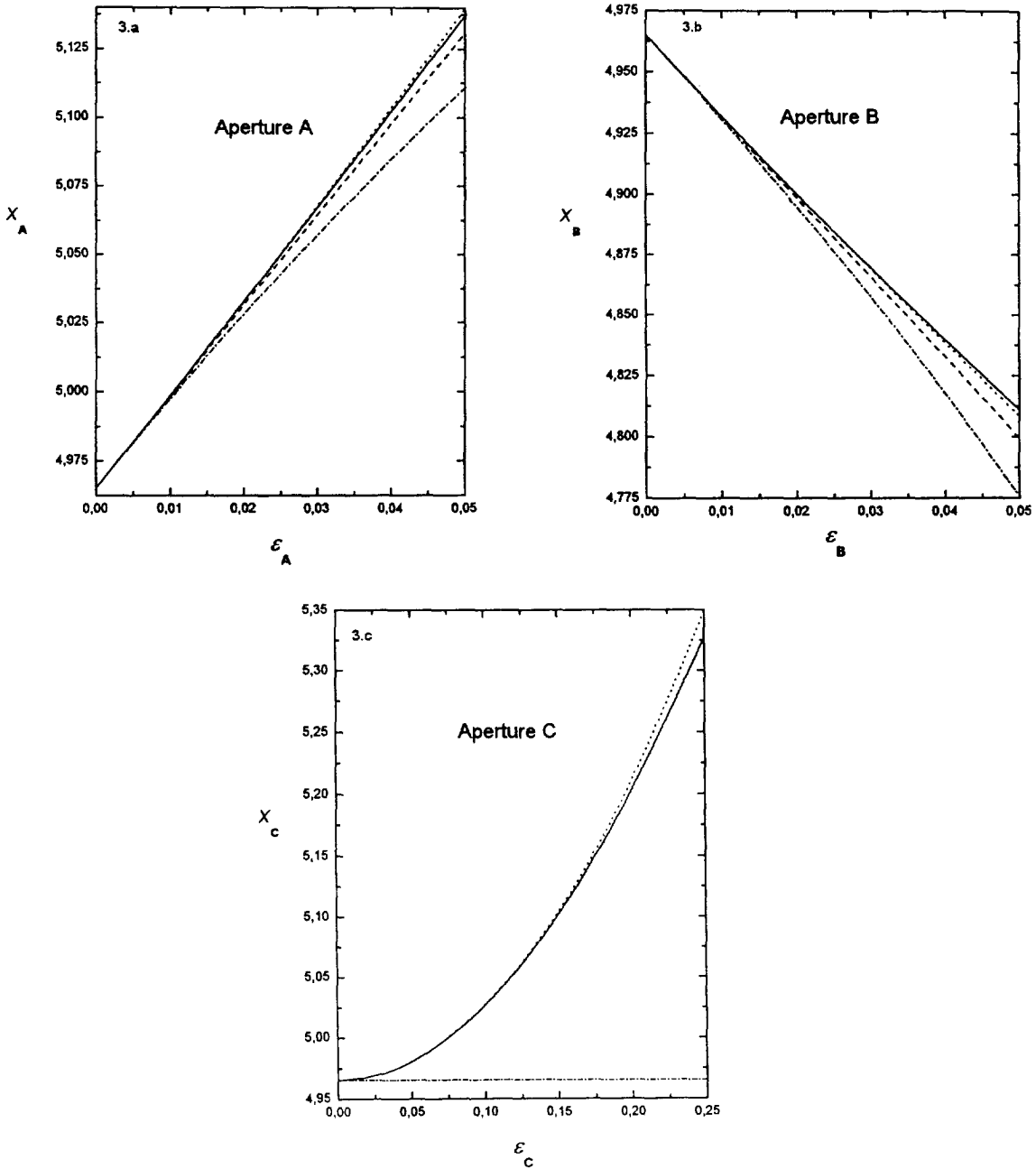


Fig. 3. First- and second-order results for the displacement law. The first-order (i.e., dashed-dotted, and dashed) curves are the same as in Fig. 2. The full lines are numerical solutions to the second-order equations (20)–(22). The dotted lines correspond to the quadratic approximations given by Eqs. (25)–(27).

first-order implicit equation (11) is $x_A = 5.1108$ and we obtain $\lambda_{\max A} = ch/kT_A x_A = 1.4076 \mu\text{m}$ (the first-order approximation (17) yields a reasonably close result, namely $\lambda_{\max A} = 1.4021 \mu\text{m}$). The second-order implicit equation (20) yields $x_A = 5.1371$ and therefore $\lambda_{\max A} = 1.4004 \mu\text{m}$ (the corresponding approximation (25) yields $\lambda_{\max A} = 1.3997 \mu\text{m}$). Thus, the first-order (or near-equilibrium diffusion) theory yields a wavelength of maximum intensity which is 2.9% less than the value predicted by the zeroth-order (or equilibrium) theory, whereas the second-order correction, also with respect to the equilibrium value, is 3.3%. Such differences are not negligible and are therefore susceptible to being measured experimentally. The simplest way would probably be, in our opinion, to make use of an enclosure such as that depicted in Fig. 1 and containing a gas that has an absorption coefficient as gray as possible near the wavelength of maximum intensity of radiation at the chosen temperature of work. Otherwise, the gray approximation may be too hard an assumption [12], and extending the method presented here in order to drop it seems to be very complicated mathematically [9].

In closing we would like to stress that there are many physical systems for which contact thermometers cannot be used. Therefore, let us further exemplify our results in the following way: assume that we want to make use of the displacement law in order to determine the temperature of a system with $\sigma = 0.1 \text{ m}^{-1}$ and $|dT/dz| = 10 \text{ K/m}$. If we make use of a spectrophotometer, find that the wavelength of maximum intensity is $\lambda_{\max A} = 1.4004 \mu\text{m}$ and simply apply the Wien displacement law, what is the error made in the temperature thus determined? The temperature thus determined is $T_A = ch/4.9651k\lambda_{\max A} = 2069 \text{ K}$. This approximate result amounts to neglecting the presence of a temperature gradient in the system. However, we have just seen that, according to the second-order theory, the temperature in this case would be $T_A = 2000 \text{ K}$. Thus, the error is 3.5%. Such an error should not be neglected. This, together with the fact that it is currently necessary to determine the temperatures of radiative systems under temperature

gradients [1–4], indicates the relevance of extending Wien's displacement law to nonequilibrium systems.

Acknowledgement

The authors are very pleased to thank D. Jou for suggesting this problem to them. Discussions with him and P. Roure are also acknowledged. This work was partially funded by the Comisión Interministerial de Ciencia y Tecnología of the Spanish Government under grant No. CLI 95-1867.

References

- [1] L.H. Aller, *Astrophysics, The Atmospheres of the Sun and Stars* (Ronald, New York, 1953).
- [2] L. Kondic, J.I. Gersten, C. Yuan, *Phys. Rev. E* 52 (1995) 4976.
- [3] J.V. Nicholas, D.R. White, *Traceable Temperatures* (Wiley, Chichester, 1994), Chapter 8.
- [4] D.P. DeWitt, G.D. Nutter, *Theory and Practice of Radiation Thermometry* (Wiley, New York, 1988).
- [5] M. Stix, *The Sun* (Springer, Berlin, 1989), Chapter 4.
- [6] K.S. Suslick, in: *Modern Synthetic Methods*, Vol. 4, ed. R. Scheffold (Springer, Berlin, 1986).
- [7] G.C. Pomraning, *The Equations of Radiation Hydrodynamics* (Pergamon, Oxford, 1973).
- [8] D. Mihalas, B.W. Mihalas, *Foundations of Radiation Hydrodynamics* (Oxford University Press, Oxford, 1984).
- [9] J. Fort, *Physica A* 243 (1977) 275.
- [10] S. Chapman, T.G. Cowling, *The Mathematical Theory of Non-uniform Gases* (Cambridge University Press, Cambridge, 1970).
- [11] J. Fort, J.E. Llebot, UdG preprint (unpublished).
- [12] S. Chandrasekhar, *Radiative Transfer* (Dover, New York, 1960).
- [13] L.C. Woods, *An Introduction to the Kinetic Theory of Gases and Magnetoplasmas* (Oxford University Press, Oxford, 1993).
- [14] P.T. Landsberg, *Thermodynamics and Statistical Mechanics* (Dover, New York, 1990).
- [15] L.D. Landau, E.M. Lifshitz, *Statistical Physics, Part 1* (Pergamon, Oxford, 1980).
- [16] N. Piskunov, *Differential and Integral Calculus* (Mir, Moscow, 1977), Section III-11.
- [17] S. Chandrasekhar, *An Introduction to the Theory of Stellar Structure* (Dover, New York, 1958).