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A simple scaling approach to Mott conductivity

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Abstract

We present a simple scaling form of Mott's formula of conductivity in case of amorphous materials in an arbitrary dimension d for a density of states which has a soft gap at the Fermi energy. We also indicate the scaling form of the conductivity at a high electric field. Previously known results can be obtained from this generalised form of the conductivity.

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1. Introduction

Amorphous materials have recently attracted considerable attention with regard to their electrical properties [1–3]. To understand the behaviour of electrical conduction of such a material, a mechanism originally due to Mott [4] has been developed. This mechanism is effective at sufficiently low temperature and is known in the literature as variable range hopping (VRH). This VRH model has been applied in various branches of condensed matter physics. In recent years, the VRH model has been invoked in metal insulator transition [5], insulating amorphous alloys [6], thin film transistors [7], inorganic compounds [8] and in mesoscopic carbon networks [9]. A variant of the VRH model known as the quasi-one-dimensional VRH model [10] has been applied to charge transport in the disordered regime of HCl-doped PAN-ES samples.

The localised states do not carry any current in the thermodynamic limit. Hence, the conduction involving localised states can only take place by means of transitions of electrons from full states to neighbouring empty states with the help of phonons. The extended states which can carry current at $T = 0$ (i.e. finite conductivity) can be distinguished from a localised one by a quantity known as the generalised inverse participation ratio (GIPR) [11]. For localised states GIPR vanishes while for extended ones GIPR is non-zero when the system size is taken to infinity. This GIPR has been recently used in a two-dimensional disordered system in a magnetic field to shed light on the nature of the energy eigenstates in the band [12].

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The hopping transition rate in this VRH model can be written as

$$p = v_0 \exp(-2\alpha R - \beta W), \tag{1}$$

where α is the inverse localisation length of exponentially localised states (at large distance) one below the Fermi energy E_F and the other above E_F, R is the typical hopping distance between the localised states, $\beta = 1/k_B T$ and W is the energy separation between the final and initial state. The attempt frequency v_0 depends on the strength of the electron–phonon coupling and the phonon density of states but almost independent of R and W.

It is evident from the above formula that the electrons will prefer to hop to more distant neighbours where the energy difference between the states is smaller. The transition probability is maximised because of the existence of a trade-off between the hopping distance R and mismatch energy W subjected to the constraints that energy window $k_B T$ is quite narrow and the localisation length α^{-1} and the Fermi energy remain constant within the narrow window. This maximisation is performed by the optimisation of the exponent $\mathscr{P} = 2\alpha R + \beta W$, where W depends on R [4].

For a constant density of states in three dimension, Mott [4] found that the conductivity which is related to the transition probability varies with temperature as

$$\sigma(T) = \sigma_0 \exp\left(-\frac{A}{T^{1/4}}\right). \tag{2}$$

The physical meaning of $\frac{1}{4}$ exponent was interpreted [3] as the reciprocal of the effective dimensionality of 4 (3 spatial and one energy). Electrons in this language hop in a four-dimensional space (x, y, z, E). A more rigorous treatment of the hopping process was discussed in terms of the percolation theory [13] and by the random resistor network model [14]. If the hopping is found to occur in a two-dimensional space, then a straightforward analysis gives the exponent $\frac{1}{3}$ instead of $\frac{1}{4}$. In general for an arbitrary dimension d, the exponent is $1/(d+1)$ [3]. All these results were obtained for a *flat* density of states and neglecting electron–electron interaction. The exponent $\frac{1}{2}$ was obtained in a dilute interacting impurity model in three dimensions [15,16]. Discussions regarding the value of the exponent take place in the literature on the subject [17,18]. The zero field result for arbitrary dimension was derived by Zabrodski [19]. Even the high-field result for three dimensions and a quadratic gap was calculated by Hill [20]. An important, and still unsolved problem, is that of the conductivity exponent in the case of a soft gap and under high electric fields. This is the problem we tackle in the present paper. A soft gap is obtained when the electron–electron Coulomb interaction is taken into account, and is more realistic than a constant density of states [4]. This is why it is relevant to analyse its effect on the conductivity exponent. The experimental data lead to different exponents for the conductivity within a temperature window [21,22] and this is an important motivation to derive more general exponents as we do in this paper.

Although we are interested in the high-field case, it will be very useful to summarise how to generalise Eq. (2) for a density of states (DOS) which has a soft gap at the Fermi energy. This result was earlier obtained by Ref. [19]. We assume a DOS as

$$N(E) \sim |E - E_F|^v. \tag{3}$$

Thus, the DOS vanishes only at $E = E_F$. This particular nature of the density of states arises when studying the localised electrons interacting via Coulomb interaction [15] at low temperature. A bound for the exponent v was also obtained as being [23]

$$v \geqslant d - 1 \tag{4}$$

for an arbitrary dimension d. With the above density of states, the unitarity condition within the energy intervals $E_F \pm W(R)$ in an arbitrary dimension d indicates that

$$\frac{2\pi^{d/2} R^d}{\Gamma(d/2+1)} \frac{W^{v+1}}{(v+1)} = 1. \tag{5}$$

It is clear from the above Eq. (5) that $W(R)$ is essentially the average energy spacing among states spatially located within a hypersphere of

radius R, and then we can see that

$$W(R) \sim \frac{1}{R^{d/(v+1)}}. \quad (6)$$

Eq. (6) can also be easily derived without the use of Eq. (5) in the following way. Note that W is proportional to $1/(V\langle N\rangle)$ [4], where V is the volume (proportional to R^d in d dimensions) and $\langle N\rangle$ is the mean number of states within the energy intervals $E_F \pm W(R)$. Now, $\langle N\rangle$ can be computed easily from Eq. (3) and in this simple way there also follows Eq. (6).

It is also evident that for $v = 0$ (flat DOS) $W(R)$ is proportional to R^{-d}. Hence, the exponent $\mathscr{P}$ in the hopping transition rate in Eq. (1), $p = v_0 \exp(-\mathscr{P})$ becomes

$$\mathscr{P} = 2\alpha R + \frac{\beta b}{R^{d/(v+1)}}. \quad (7)$$

Here b is a constant independent of R, W and T. Maximising the hopping transition rate with respect to R we obtain

$$\sigma(T) = \sigma_0 \exp\left(-\frac{A}{T^{\phi}}\right), \quad (8)$$

where the exponent ϕ is given by

$$\phi = \frac{v+1}{d+v+1}, \quad (9)$$

which matches with an earlier result [19]. The route to the previously known Eq. (9) will now be very useful to consider the case we are interested in, namely an arbitrary gap and number of dimensions in a high field.

2. Extension to high electric field

The above result is valid for low temperatures and weak electric fields. For a weak electric field (F), the electric field-dependent conductivity [1] is given by

$$\sigma(F) \sim \exp(eRF/k_B T), \quad (10)$$

where R is the most probable hopping distance.

Experimentally, this low-field conductivity dependence on electric field and temperature has been observed [24] in P-doped Si samples. Extending this result, we next show that at sufficiently high electric field intensities there is no temperature dependence of the conductivity. The effect of an applied field $\vec{F}$ is to modify the energy of a hop between the two states separated by a distance R by an amount $eFR\cos\theta$, where θ is the angle between the direction of $\vec{R}$ and $\vec{F}$ and e is the electronic charge. At high field and for $\vec{R}$ in the direction of $\vec{F}$ (i.e. $\cos\theta \approx 1$), the quantity eFR becomes higher than the average hopping energy $W(R)$ and hence, electrons can hop without the need of energy transfer from the phonons, so that the transition probability p will contain $v_0 \exp(-2\alpha R)$. Since, in the strong field limit, the thermal energy of the phonons does not contribute to the hopping mechanism, no temperature dependence of the conductivity is expected. To find the scaling form in this situation, we note that the Eq. (5) becomes

$$\frac{2\pi^{d/2} R^d}{\Gamma(d/2+1)} \frac{(eFR)^{v+1}}{(v+1)} = 1. \quad (11)$$

Therefore, the conductivity will take the form

$$\sigma(T) = \sigma^{HF} \exp\left(-\frac{B}{F^{\phi}}\right), \quad \phi = \frac{v+1}{d+v+1}. \quad (12)$$

Previously known results [20,25,26] can be obtained from this generalised form of the conductivity in high electric fields. In the special case of three-dimensional systems and a quadratic DOS ($v = 2$), we recover the familiar results of Refs. [19,20].

In summary, we have obtained a generalised form for conductivity in an arbitrary dimension d with a density of states having a soft gap in amorphous materials. This formula can be applied to an interacting system. At a high electric field, the scaling form of the conductivity with the applied electric field has been found as given by Eq. (12).

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