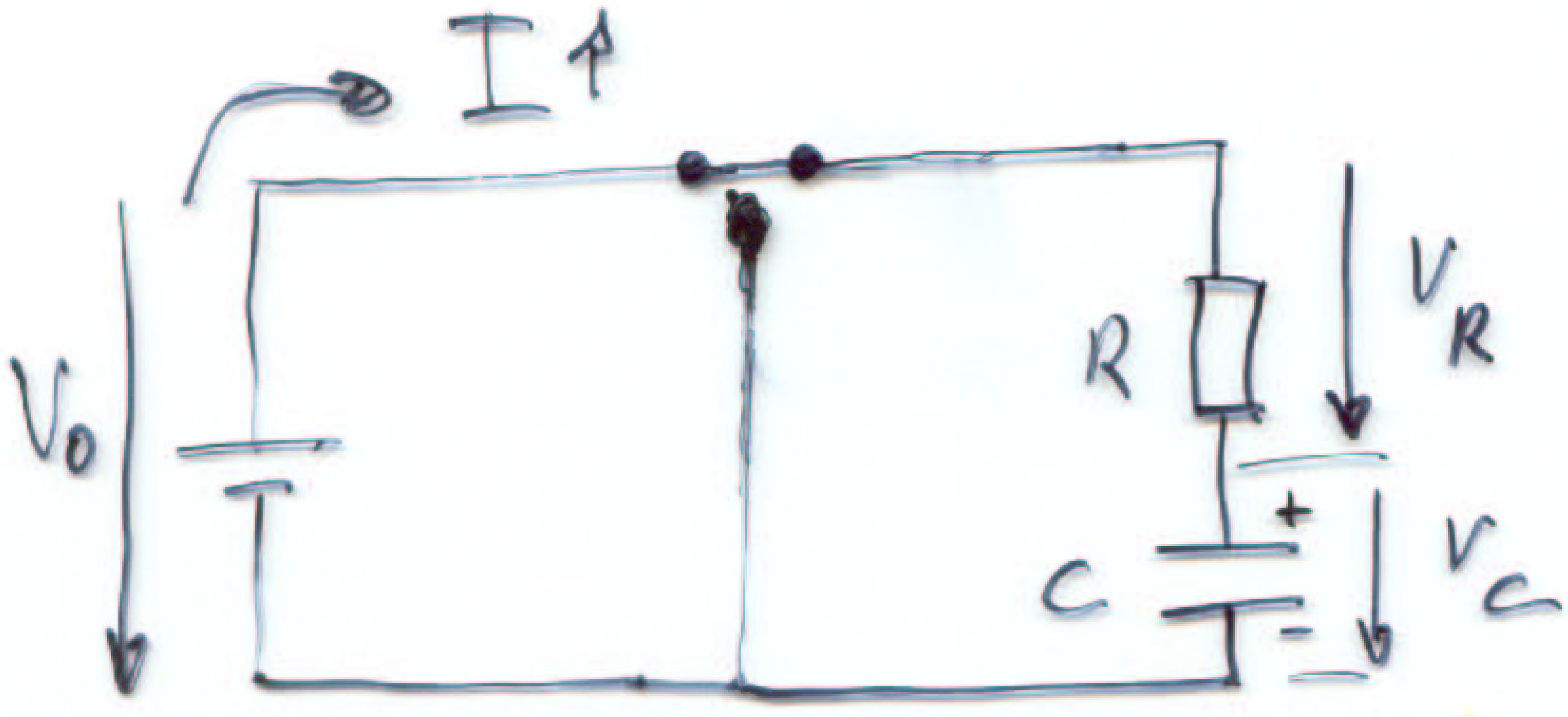
CARGA Y DESCARGA DE UN CONDENSADOR. CIRCUITO RC

CARGA.



$$V_0 = V_R + V_C$$

$$V_R = IR = R\frac{dq}{dt}$$

$$V_C = \frac{q}{C}$$

$$V_0 = R\frac{dq}{dt} + \frac{q}{C}$$

multiplico por C:

$$V_0 C = RC\frac{dq}{dt} + q$$

$$V_0 C - q = RC\frac{dq}{dt}$$

$$\frac{dt}{RC} = \frac{dq}{V_0 C - q} \rightarrow \int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{CV_0 - q}$$

$$\frac{t}{RC} = \Big[-\ln\left(CV_0 - q\right)\Big]_0^q$$

$$-\frac{t}{RC} = \ln\left(CV_0 - q\right) - \ln CV_0 = \ln\frac{CV_0 - q}{CV_0}$$

$$\frac{CV_0 - q}{CV_0} = e^{-t/RC}$$

$$CV_0 - q = CV_0\, e^{-t/RC}$$

$$-q = CV_0\left(e^{-t/RC} - 1\right)$$

$$q = CV_0\left(1 - e^{-t/RC}\right)$$

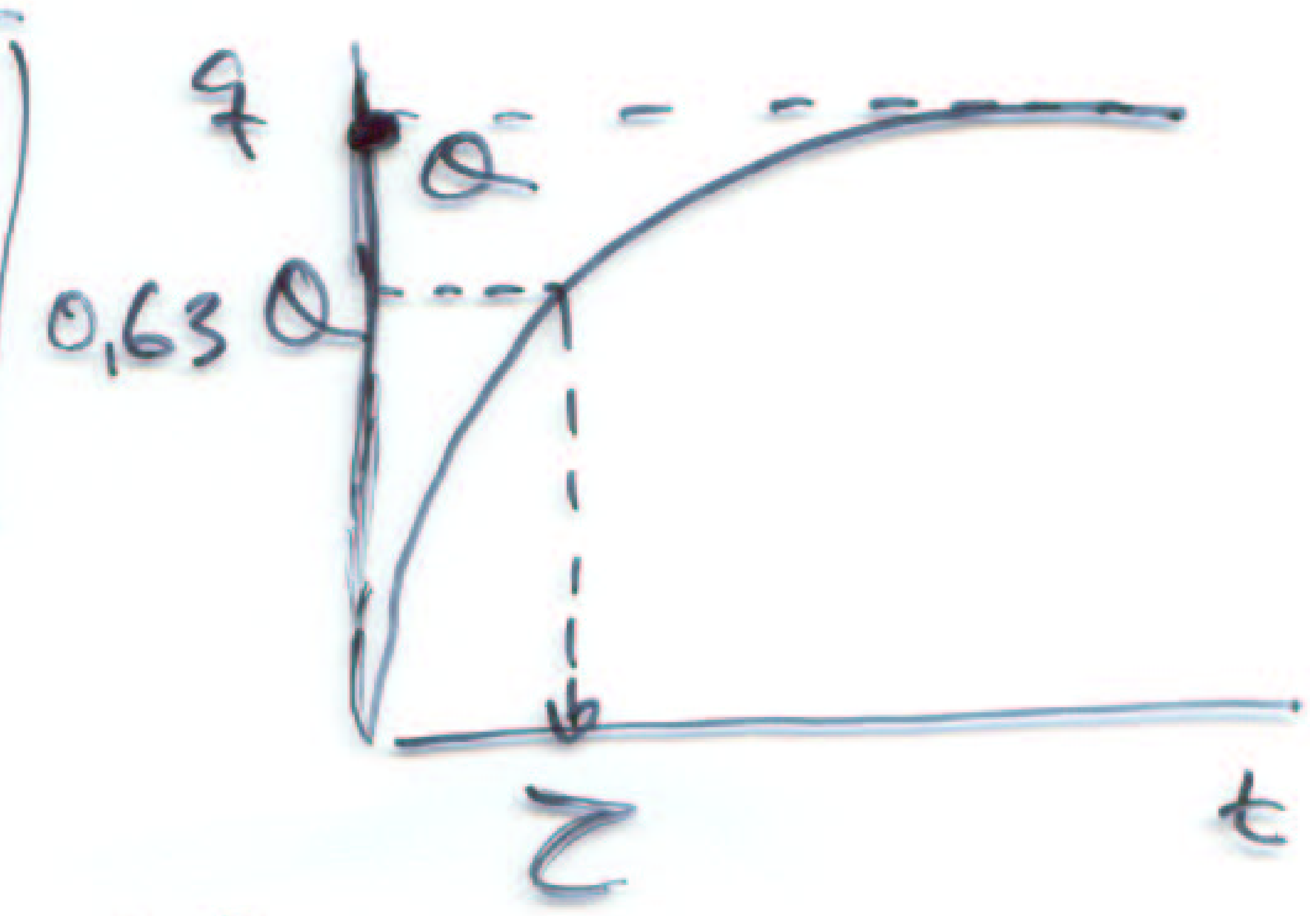
$$Q \equiv CV_0$$

$$RC = \tau$$

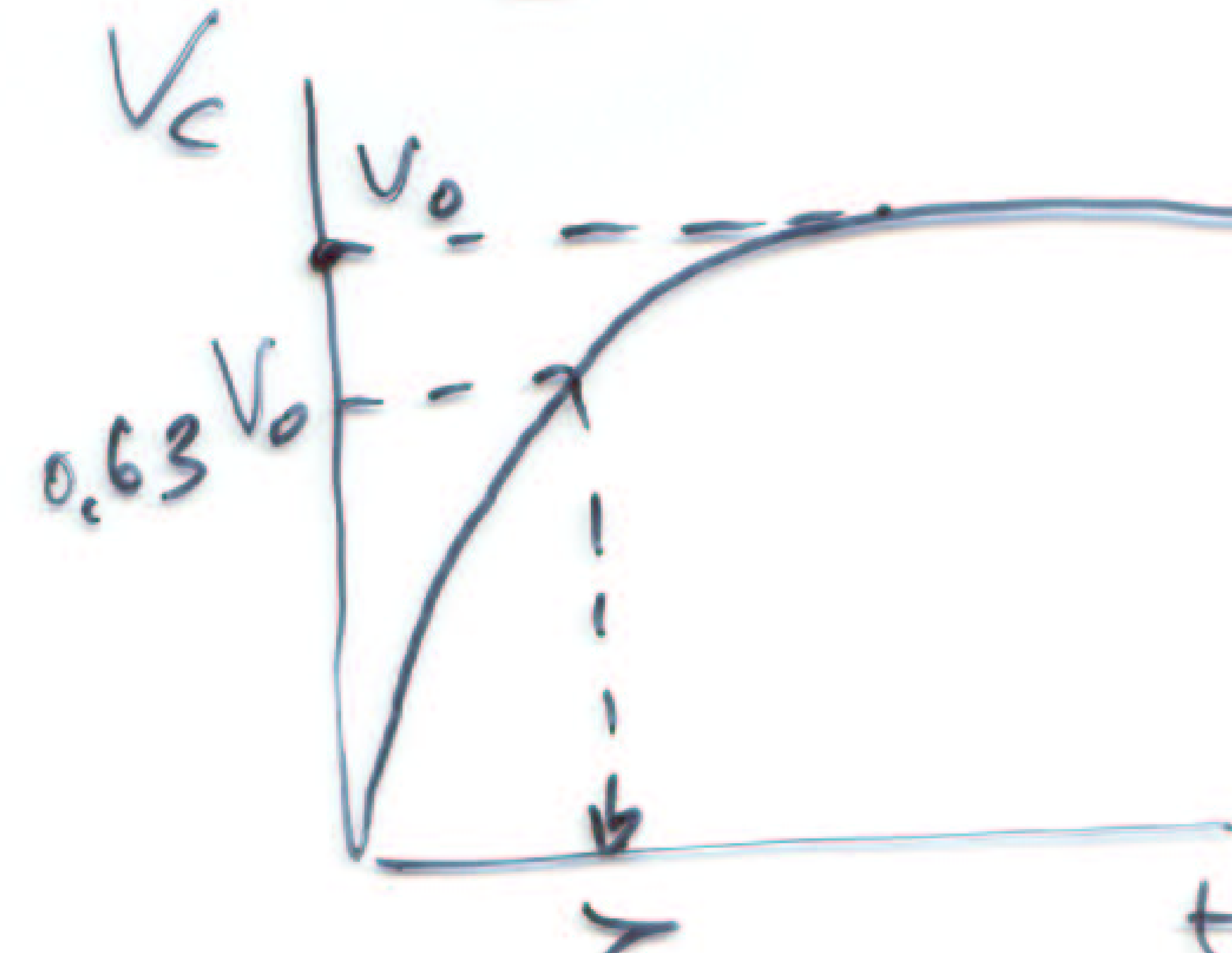
↳ constante de tiempo

$[RC] = T$; SI → s.

$$\boxed{q(t) = Q\left(1 - e^{-t/RC}\right)}$$

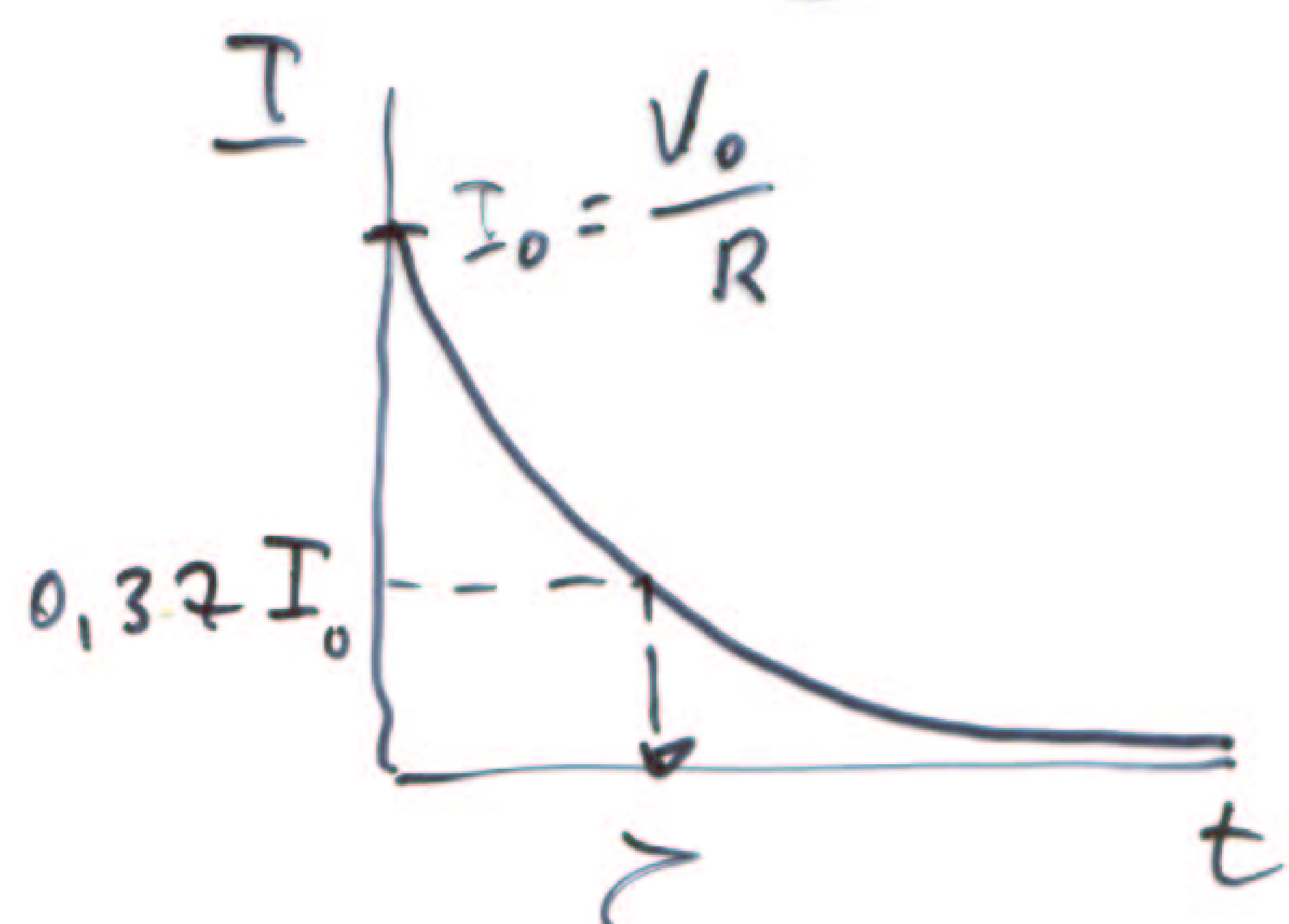


$$\boxed{V_c(t) = V_0\left(1 - e^{-t/RC}\right)}$$



$$I(t) = \frac{dq}{dt}$$

$$\boxed{I(t) = \frac{V_0}{R}\, e^{-t/RC}}$$



$$E_{bateria} = Q V_0 = C V_0^2$$

↳ trabajo realizado por la bateria durante el proceso de carga

→ Cuando el condensador se ha cargado

$$E = \frac{1}{2} Q V_0 = \frac{1}{2} C V_0^2$$

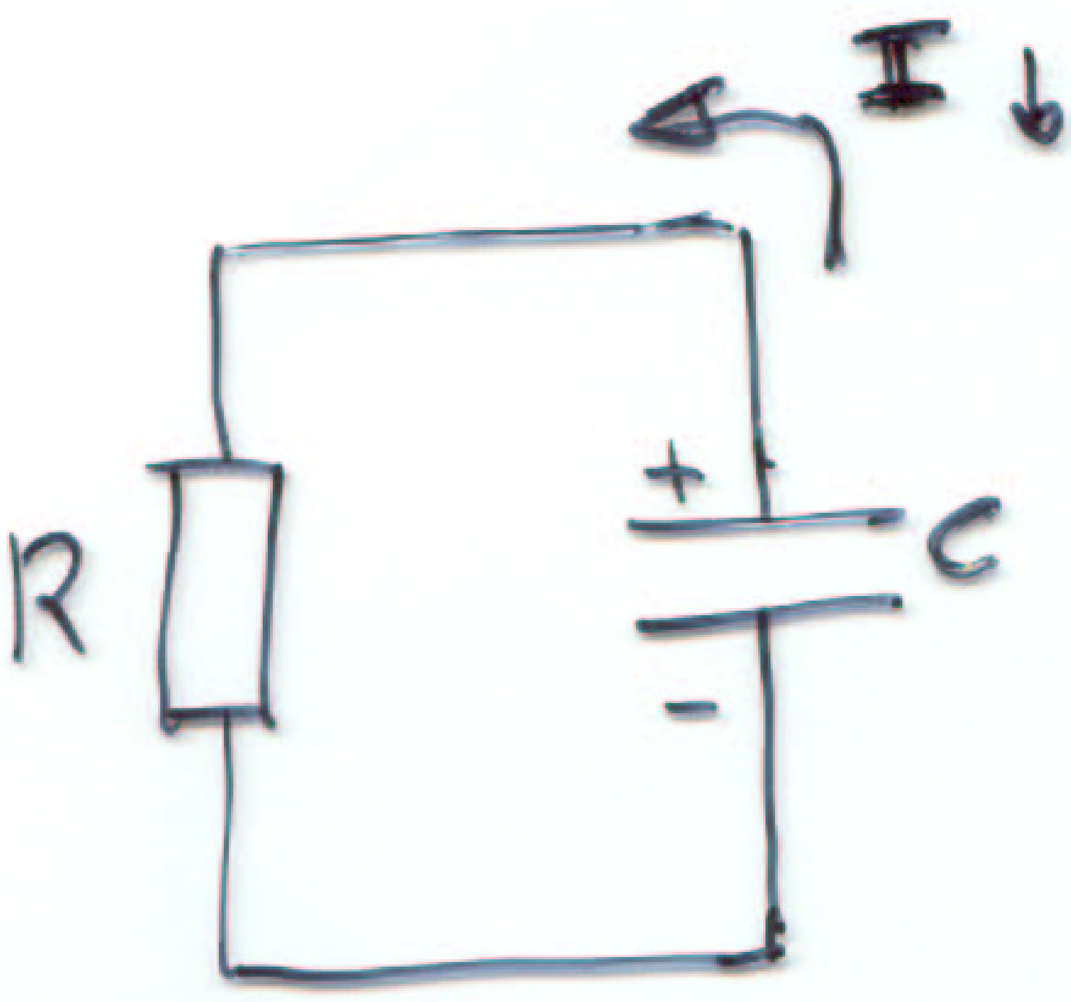
↳ energia almacenada en el condensador

$E_{bateria}$ ↗ se almacena en el condensador
↘ se disipa en forma de calor en la resistencia

En Resumen:

en $t = 0$ $\left\{ \begin{array}{l} q = 0 \\ V_0 = 0 \\ I = \dfrac{V_0}{R}. \end{array} \right.$ $\qquad t \to \infty$ $\left| \begin{array}{l} q \to C V_0 \\ V_C \to V_0 \\ I \to 0 \end{array} \right.$

DESCARGA



En $t=0 \rightarrow$ carga $= Q$

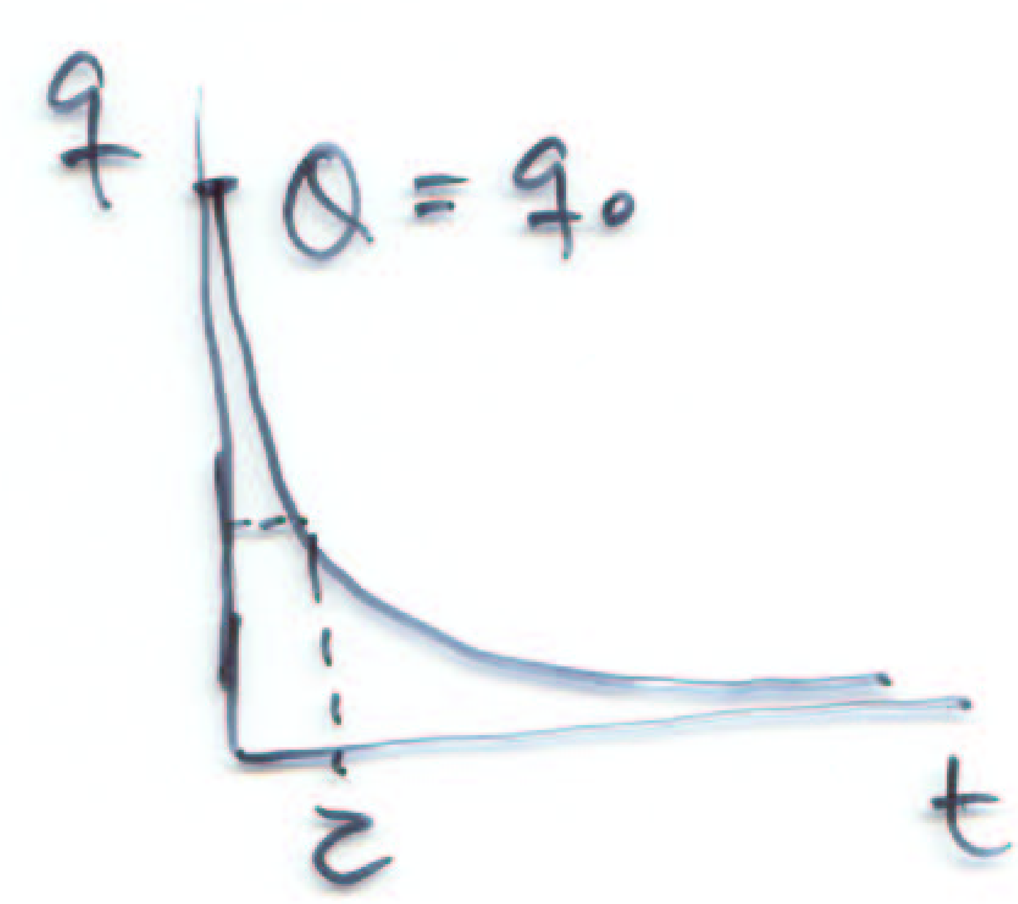
en un instante t :

$$\left.\begin{aligned} V_c - IR &= 0 \\ I &= -\frac{dq}{dt} \\ V_c &= \frac{q}{C} \end{aligned}\right|$$

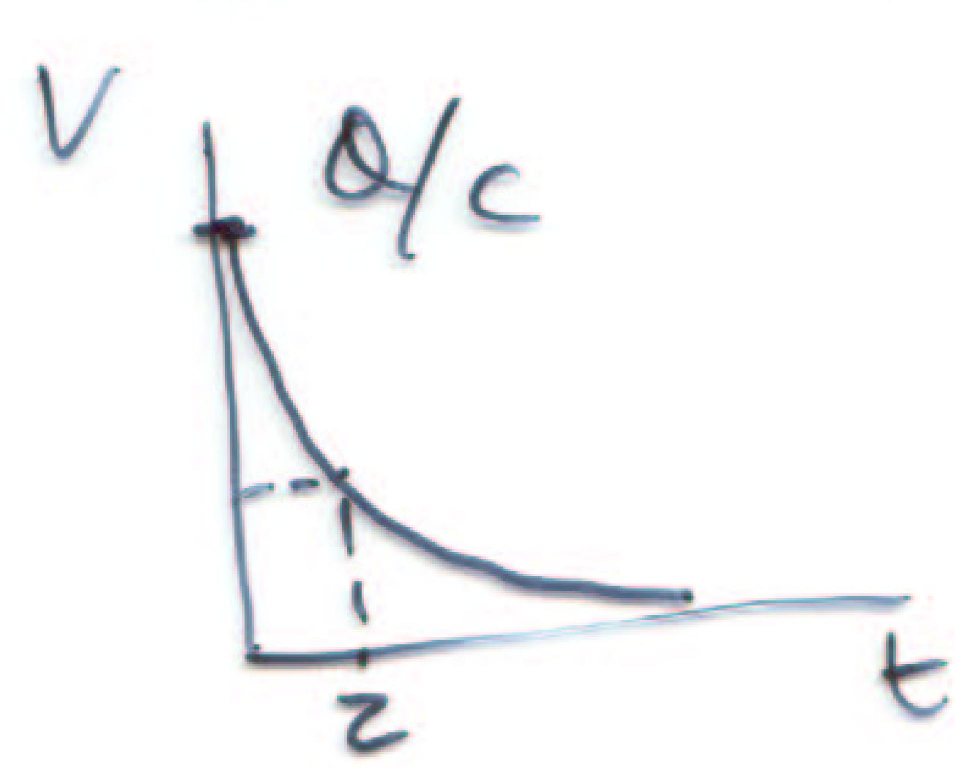
$$R\,\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\int_{q_0}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

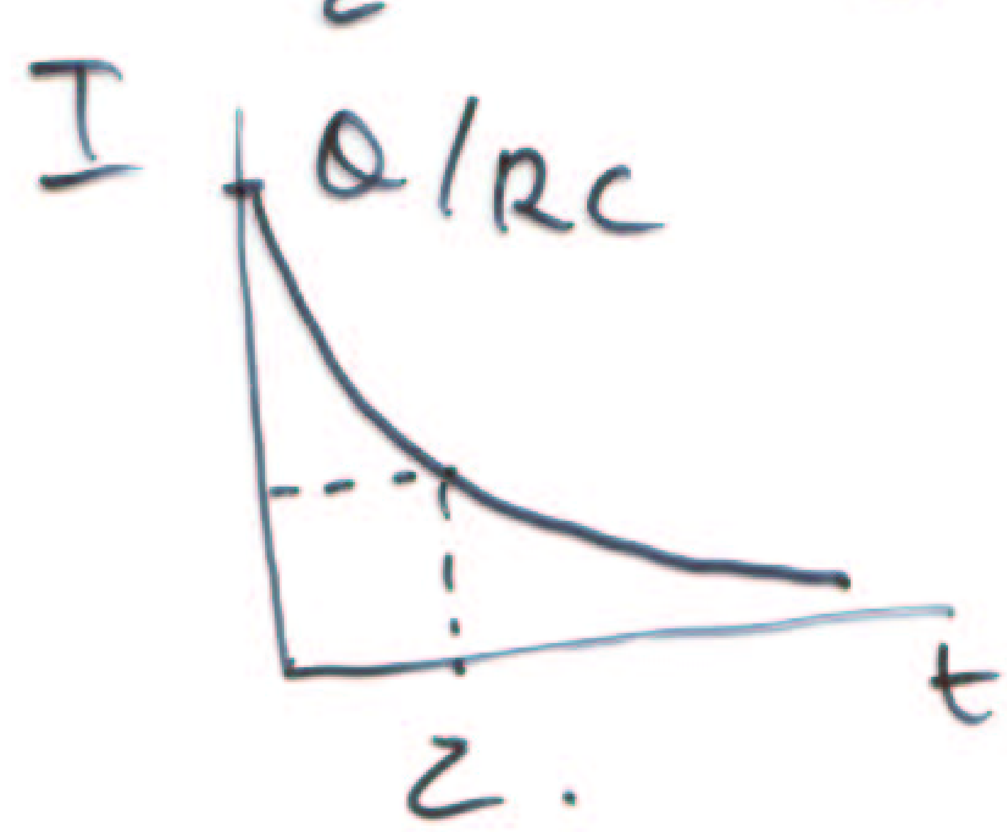
$$q(t) = \underset{\overset{\shortparallel}{Q}}{q_0}\, e^{-t/RC}$$



$$V_c(t) = \frac{q_0}{C}\, e^{-t/RC}$$



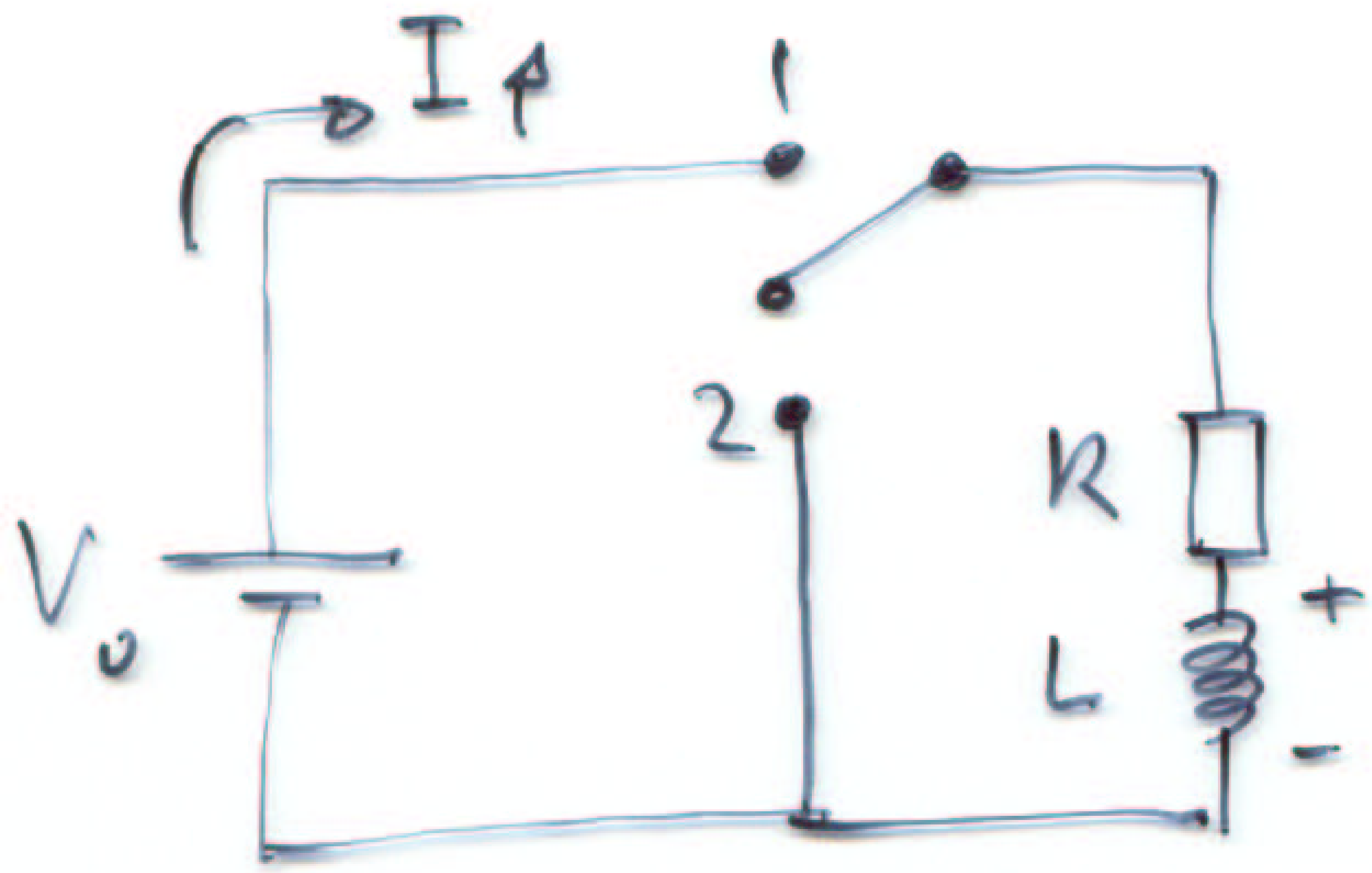
$$I(t) = \frac{q_0}{RC}\, e^{-t/RC}$$



CONEXIÓN Y DESCONEXION EN UN CIRCUITO RL

4bis

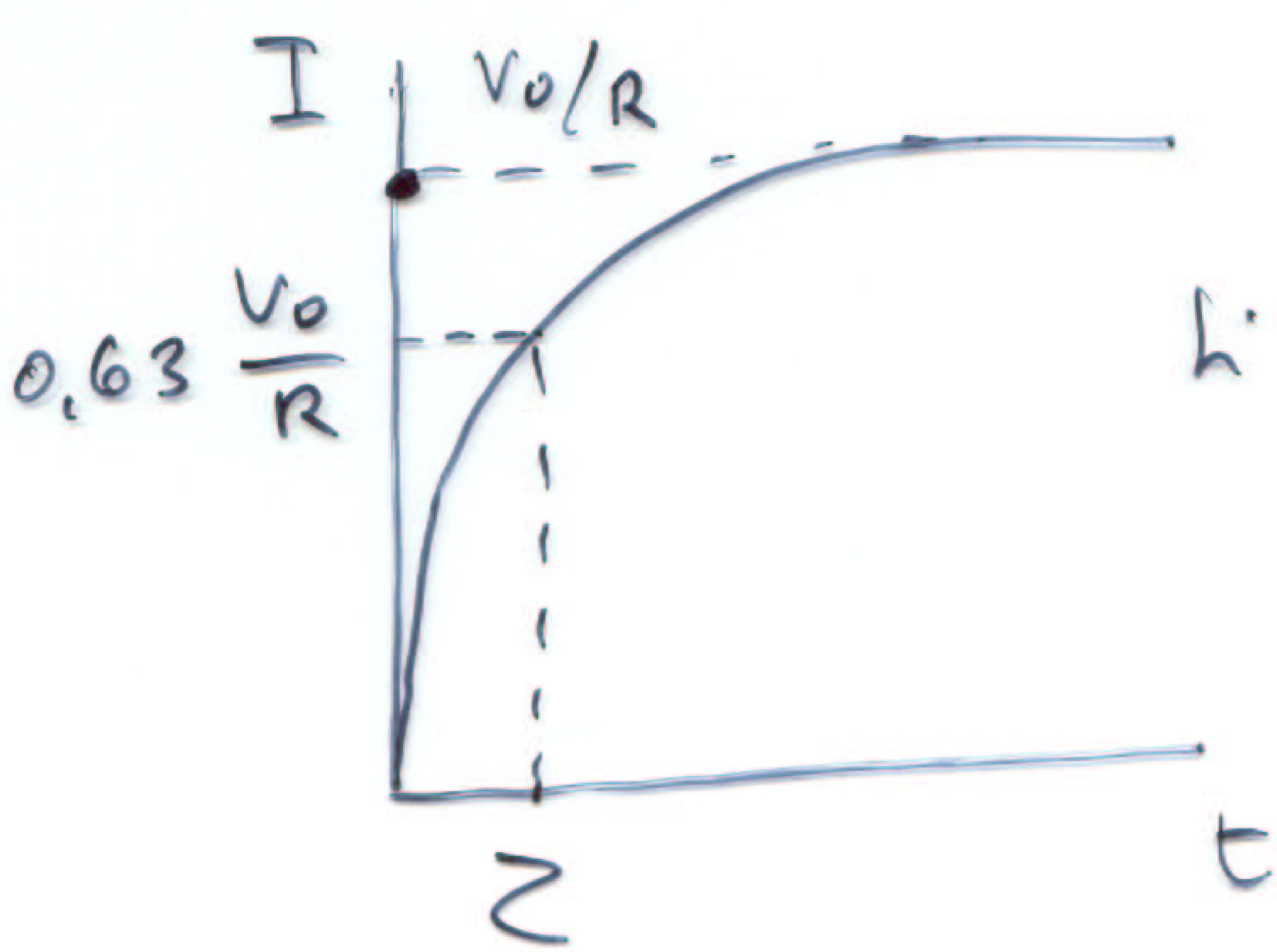
CONEXIÓN



$$V_0 - IR - L\frac{dI}{dt} = 0$$

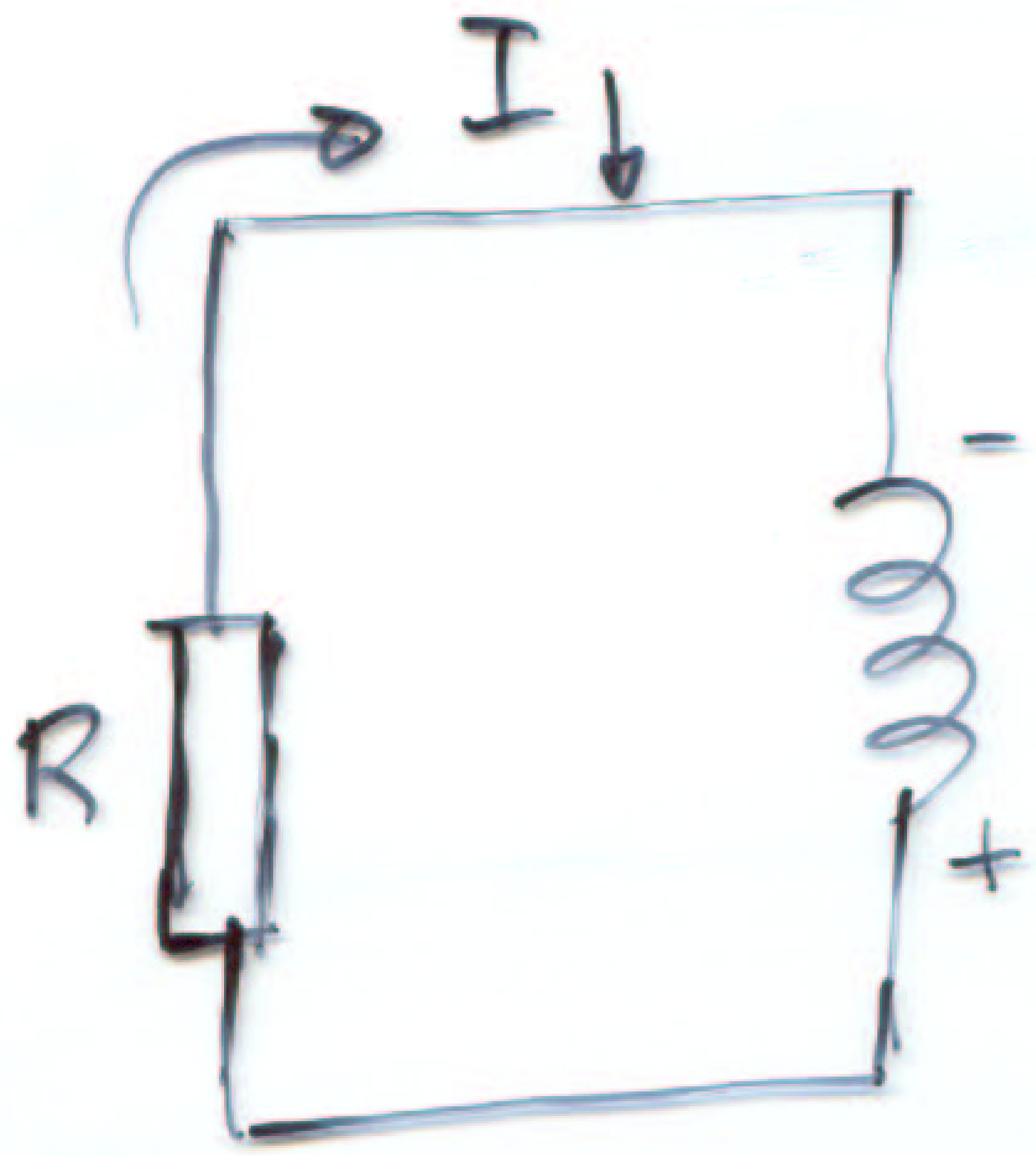
$$V_0 = IR + L\frac{dI}{dt}$$

$$\boxed{I = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad \tau = \frac{L}{R} \text{ (cte de tiempo)}}$$



si $t \to \infty$ $\quad I \to \frac{V_0}{R}$

DESCONEXIÓN



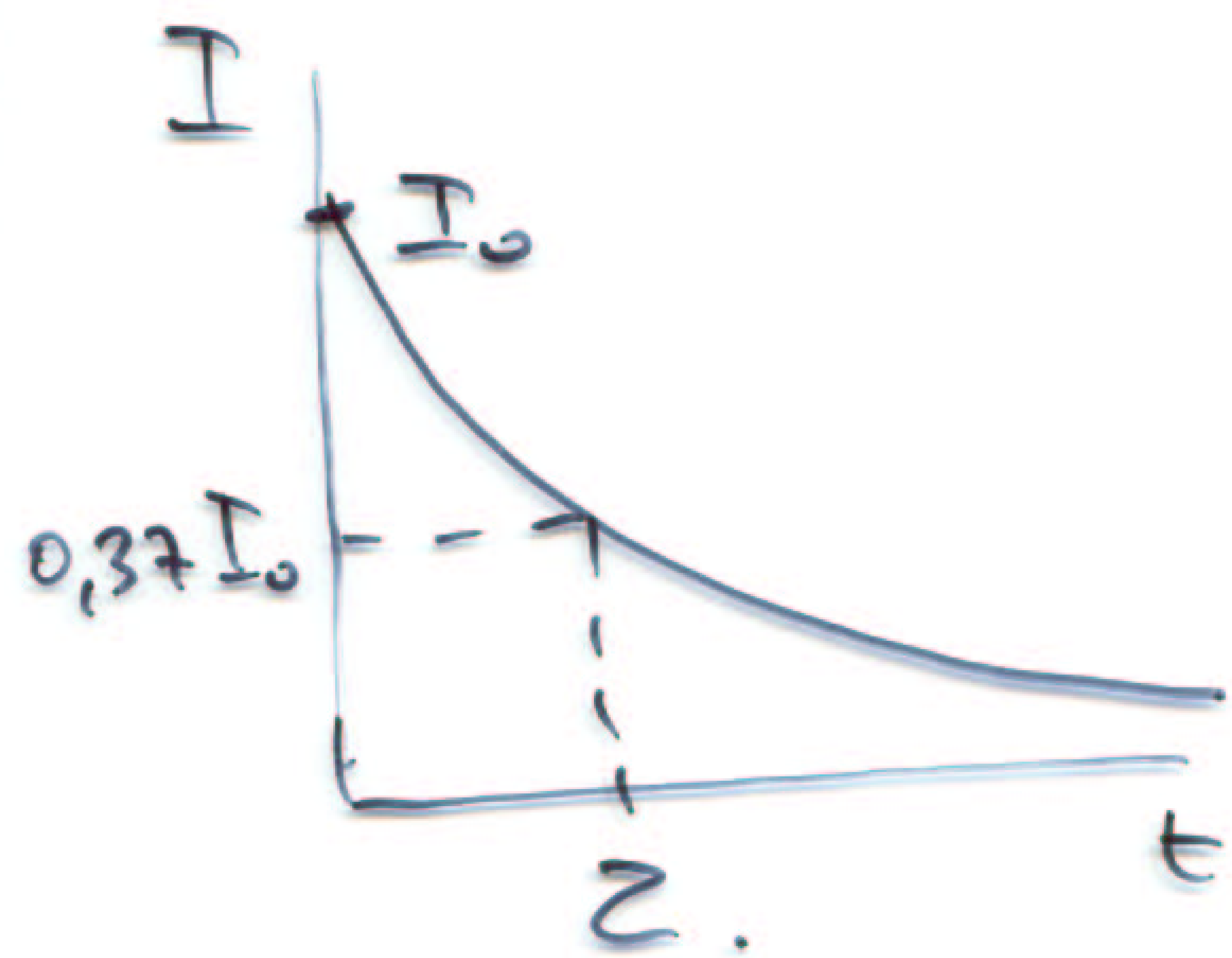
$$-IR + L\frac{dI}{dt} = 0$$

Como $I \downarrow$ al $\uparrow t$

$$-IR - L\frac{dI}{dt} = 0$$

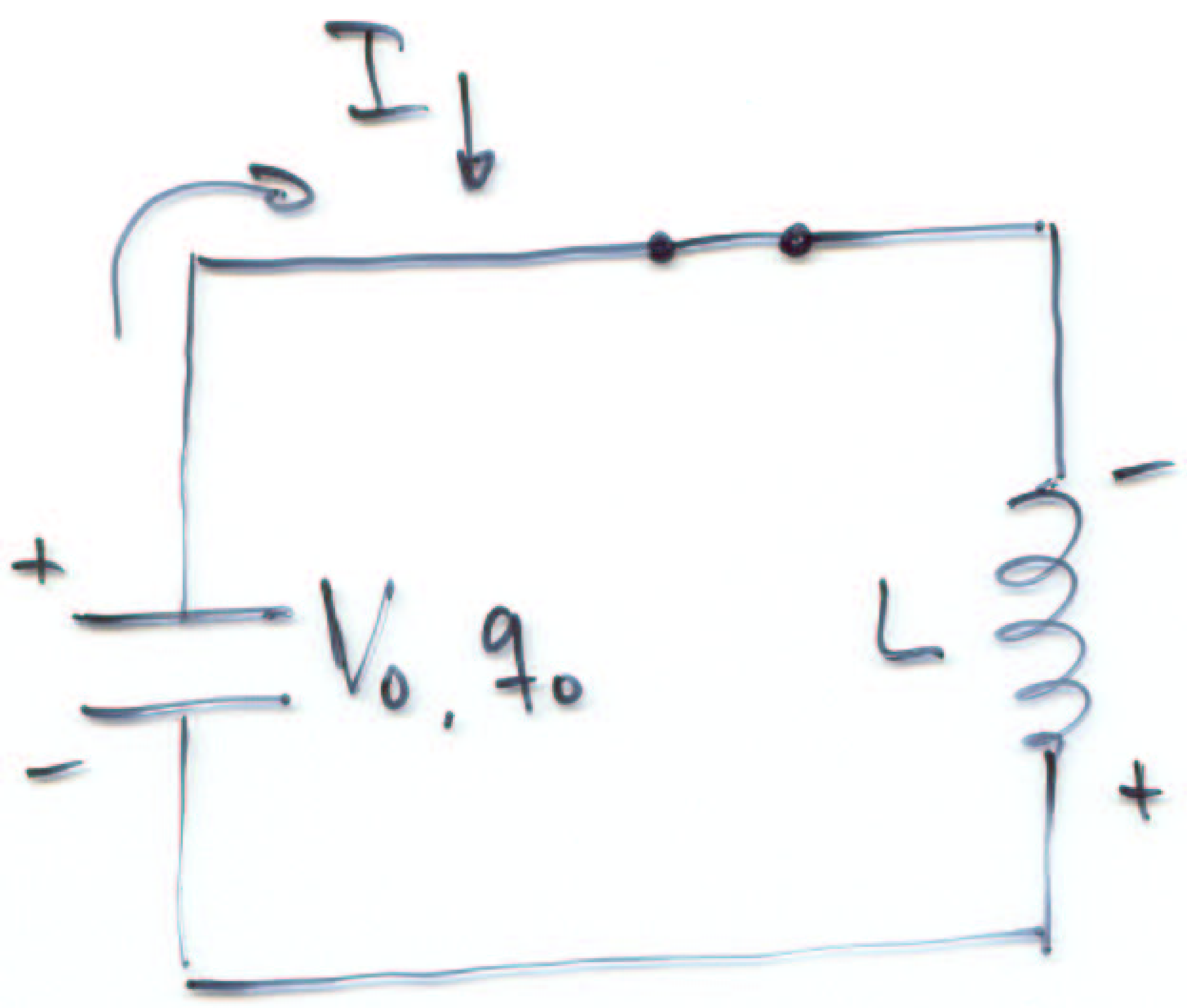
$$IR = -L\frac{dI}{dt}$$

$$\boxed{\begin{gathered} I(t) = I_0 e^{-t/\tau} \\ I_0 = \frac{V_0}{R} \\ \tau = \frac{L}{R}. \end{gathered}}$$



CIRCUITO LC ⇒

⇒ Oscilaciones eléctricas libres



en $t=0 \Rightarrow q = q_0 \equiv Q$

en un instante t:

$$V_c = \frac{q}{c}$$

$$I = -\frac{dq}{dt}$$

↳ porque disminuye con el tiempo

$$\frac{q}{c} + L\frac{d^2q}{dt^2} = 0$$

$$\boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0.}$$

Ec. diferencial de 2º orden - homogenea

Solución de esta ecuacion:

$q(t) = q_m \operatorname{sen}(\omega t + \delta) \rightarrow$ Sol. armónica

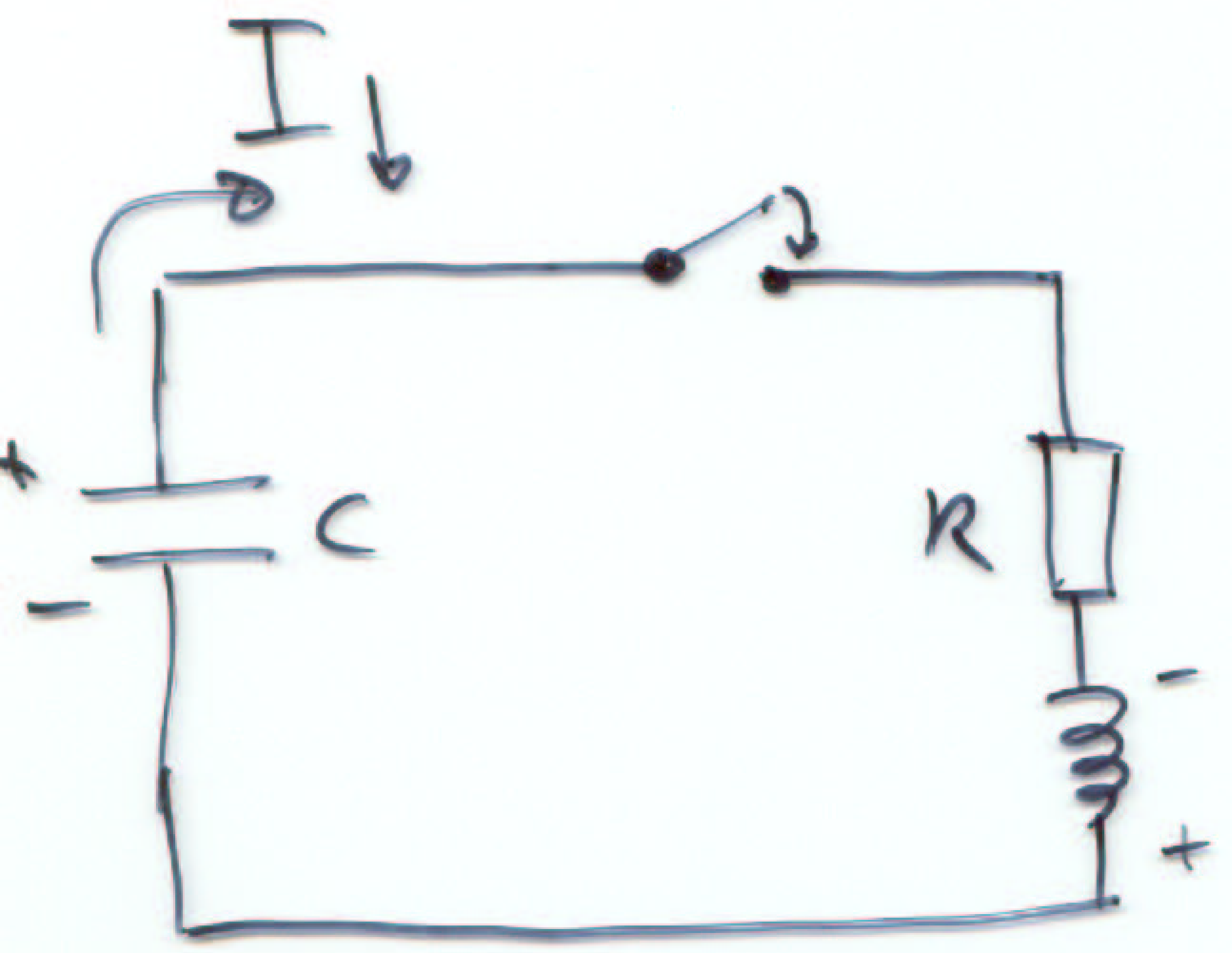
$\omega \equiv \dfrac{1}{\sqrt{LC}} \rightarrow$ pulsación (cte característica del circuito)

$$\left.\begin{array}{l}\delta = \text{fase} \\ q_m = \text{carga máxima}\end{array}\right\} f(q_0, I_0)$$

condiciones iniciales

CIRCUITO RLC →

→ Oscilaciones eléctricas amortiguadas



$$V_C - V_R + \mathcal{E}_L = 0$$

$$V_C = \frac{q}{C}$$

$$V_R = I R$$

$$\mathcal{E}_L = -L \frac{dI}{dt}$$

$$I = -\frac{dq}{dt}$$

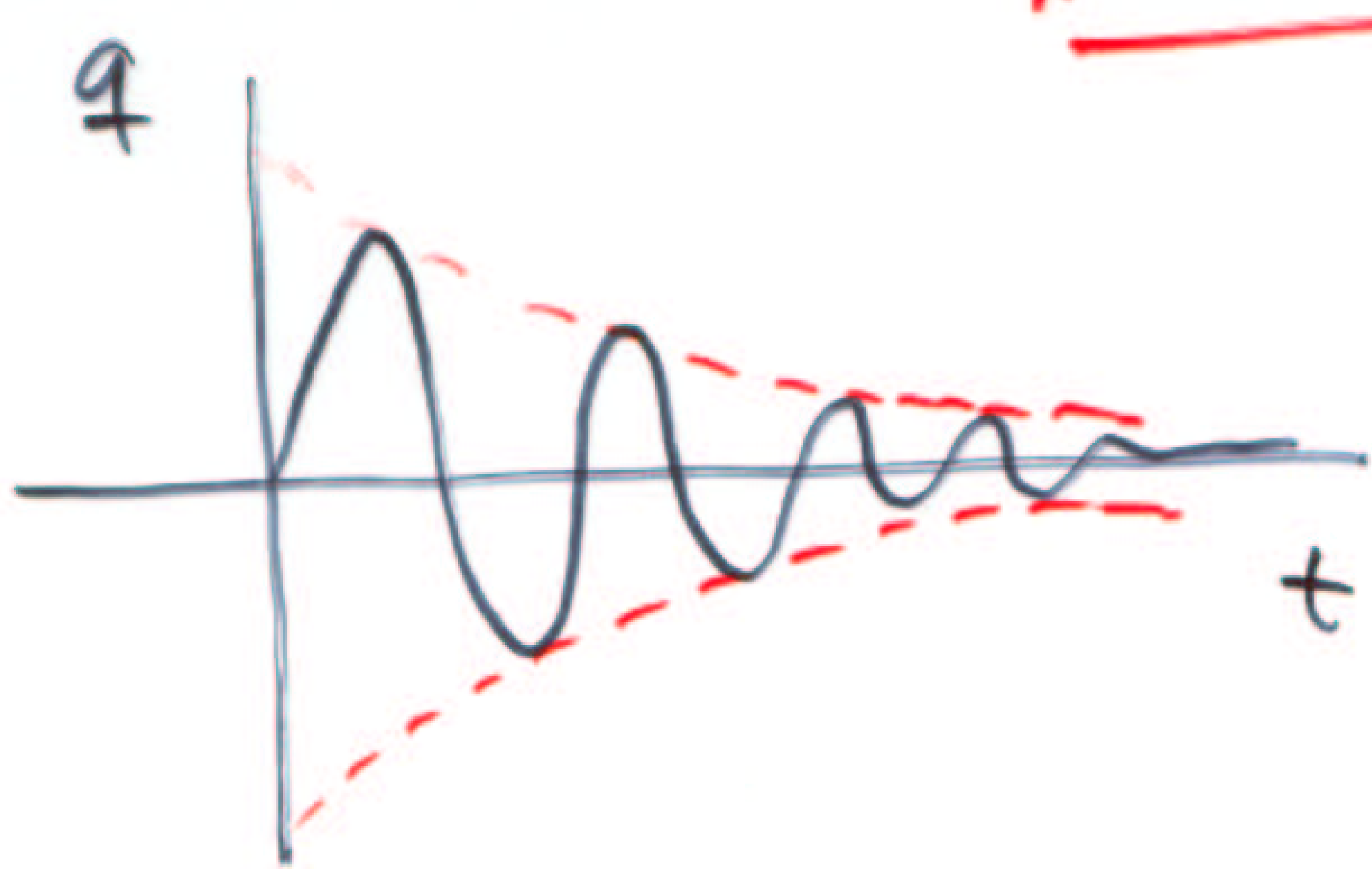
$$\frac{q}{C} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = 0$$

$$\boxed{\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0}$$

Ec. diferencial de 2º orden

Solución : oscilatoria

débilmente amortiguada.



$$\boxed{\ddot{q} + 2\gamma\dot{q} + \omega_0^2 q = 0}$$

donde $\gamma = \dfrac{R}{2L}$ cte de amortiguamiento

$$\omega_0 = \sqrt{\frac{1}{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC}$$

Solución de la ec. diferencial:

$$q(t) = q_m e^{-\overbrace{\frac{R}{2L}}^{\gamma} t} \operatorname{sen}(\omega' t + \delta)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega_0^2 - \gamma^2}$$

↳ cte del circuito
pseudopulsación.

$$\left.\begin{array}{l}\delta = \text{fase} \\ q_m = \text{carga máxima}\end{array}\right\} f\left(\begin{array}{c}\text{condiciones} \\ \text{iniciales}\end{array}\right)$$

$q_0,\ I_0$

* Criticamente amortiguado

$$\omega_0 = \gamma \rightarrow R = \sqrt{\frac{4L}{C}}$$

$$q(t) = (A + Bt)\, e^{-\gamma t}$$

- No oscila
- Se acerca rápidamente al equilibrio

* Sobreamortiago

$$\omega_0 < \gamma \rightarrow R > \sqrt{\frac{4L}{C}}$$

- No oscila

$$q(t) = e^{-\gamma t}\left[A e^{-\omega t} + B e^{-\omega t}\right]$$